EVALUATIVE CRITERIA FOR AGGREGATION RULES IN THE ANALYTIC HIERARCHY PROCESS

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The Analytic Hierarchy Process (AHP) has been considerably criticized for its possible rank reversal phenomenon. Many variants of the original AHP have been proposed to preserve rank orders. The validity of the original AHP and its variants have been a subject of long-lasting debate. In this paper we propose four criteria to evaluate the validity of AHP aggregation rules and examine three typical aggregation rules against the proposed criteria. Our research indicates that only Multiplicative AHP satisfies all of the four criteria.

Keywords: Decision analysis; Analytic Hierarchy Process; Evaluative criteria; Aggregation rules

1. Introduction

The analytic hierarchy process (AHP), developed by Thomas L. Saaty (Saaty1980), is one of the most widely used multi-criteria decision making (MCDM) techniques in decision-making field. It has been used to address many different kinds of environmental (Xiong et al., 2007; Handfeild et al., 2002), business (Wei et al., 2005; Mikhailov and Tsvetinov, 2004), medical (Liberatore and Nydick, 2008) and social (Chou et al., 2008; Srdjevic, 2007) decision-making problems (DMPs). However from the early days it has been considerably criticized for its possible rank reversal (RR) phenomenon (Wang and Elhag, 2006; Schenkerman, 1994, 1997), which means after an alternative is added or deleted the relative rankings of the other alternatives may change. Belton and Gear first noticed that when copies (or near copies) of existing alternatives are introduced in a DMP, it is possible for the AHP to change the relative rankings of the other alternatives (Belton and Gear, 1983). They attributed this phenomenon to the fact that in the AHP the values of relative performance of the alternatives in terms of each decision criterion in the decision matrix are normalized so they add up to 1.00. They proposed that these values be

Table 1. Decision matrix

Alternative	Criteria				Combined Priorities
	C_1	C_2	...	C_m	P
	(c_1	c_2	...	c_m)	
A_1	a_{11}	a_{12}	...	a_{1m}	p_1
A_2	a_{21}	a_{22}	...	a_{2m}	p_2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
A_n	a_{n1}	a_{n2}	...	a_{nm}	p_n

normalized by dividing them by the largest entry of each column of the decision matrix and argued in this manner RR does not occur. Now this variant of the original AHP is called the ideal mode AHP (Saaty, 1999). However other researchers pointed out that RR still occurs in the ideal mode AHP (Saaty and Vargas, 1984; Triantaphyllou, 2001), and they introduced several other variants of the original AHP to preserve rank orders. Schoner and Wedley presented a referenced AHP to avoid RR, which requires the modification of criteria weights when an alternative is added or deleted (Schoner and Wedley, 1989, 1992, 1993). Lootsma proposed mulitplicative AHP to replace the weighted additive aggregation rule used in the original AHP (Lootsma, 1993). Barzilai and Golany proved that no normalization could prevent RR and suggested using weighted-geometric-mean aggregation rule to avoid RR (Barzilai and Golany, 1994). Triantaphyllou offered two new cases to demonstrate that RR does not occur with the multiplicative AHP, but do occur with the original AHP and the ideal mode AHP (Triantaphyllou, 2001). He also argued if RR is supposed to be avoided when using additive aggregation rules, the rankings of alternatives should be derived by decomposing DMP into a set of smaller sub-problems and combining the partial solutions of these sub-problems. Y.-M. Wang and T. M. S. Elhag recently wrote another paper to overcome RR in AHP which also gave an excellent review on RR issue (Wang and Elhag, 2006).

Essentially RR correlates with the critical issue of MCDM, that is how to combine the relative performances of each alternative in terms of each criterion to derive proper overall alternative rankings. Suppose decision matrix is shown as Table 1, in which a_{ij} denotes the value of relative performance of A_i $(i = 1, ..., n)$ with respect to criterion C_j $(j = 1, ..., m)$; c_j denotes the contribution of C_j to decision goal and $\sum_{j=1}^{m} c_j = 1$; P is priority vector in which p_i denotes the value of the relative priority of alternative A_i combined with all the decision criteria. If $p_i > p_j$, it is assumed $A_i \succ A_j$, which means A_i is superior to A_j.

If there's no constraint, it's easy to see there are numerous aggregation rules to derive a priority vector from a given decision matrix. The critical questions are which aggregation rule is valid and how to judge the validity. Unfortunately

Table 2. Example 1

Alternative	Criteria		
	C_1	C_2	C_3
	(0.3	0.1	0.6)
A_1	1	3	1
A_2	2	1	1

Table 3. Example 2

Alternative	Criteria		
	C_1	C_2	C_3
	(0.6	0.2	0.2)
A_1	8	6	7
A_2	2	3	1

these questions may not have correct answers because in many MCDM problems even if the relative performances of alternatives with respect to all criteria are known, there is still no dominant alternative. For example, suppose a decision matrix is shown as Table 2. Though $a_{12}/a_{22} > a_{21}/a_{11}$, as $c_1 > c_2$ it is still not clear which alternative should be preferred when combined with all the decision criteria. However in some cases the rankings of alternatives are obvious. For example suppose a decision table is shown as Table 3. As $a_{1k} > a_{2k}$ $(k = 1, 2, 3)$, which means A_1 is superior to A_2 in terms of all criteria, it is supposed a valid aggregation rule should give $A_1 \succ A_2$ when combined with all the criteria. If an aggregation rule gives $A_2 \succ A_1$ in this case, it can be deemed as invalid. In other words this example can be used as a measure to evaluate the validity of aggregation rules. Actually a lot of such evaluative examples exist. Synthesizing these examples in this paper we introduce four criteria to evaluate the validity of aggregation rules in AHP. We examined three typical aggregation rules and found that only Multiplicative AHP satisfies all of the four criteria. However as it is still arguable whether a valid aggregation rule should satisfy all of the four criteria, we think it is still very early to claim that aggregation rules not satisfying the proposed criteria are invalid. The rest of the paper is organized as follws: in Sec. 2 we introduce four evaluative criteria to examine the validity of aggregation rules; section 3 reviews three typical AHP aggregation rules; in Sec. 4 we evaluate the three typical aggregation rules against the proposed evaluative criteria and present three theorems; section 5 closes the paper with conclusions.

2. Evaluative Criteria for Aggregation Rules

Suppose decision matrix is described with Table 1. We introduce four criteria to examine aggregation rules in AHP.

Criterion 1: Dominance

Given two alternatives A_i and A_j, if with respect to every criterion C_k ($k = 1, 2, ..., m$), $a_{ik} \geq a_{jk}$, then $A_i \geq A_j$ (which means A_i is not inferior to A_j). Furthermore if there's at least one criterion C_r such that $a_{ir} > a_{jr}$, then $A_i \succ A_j$, otherwise $A_i = A_j$.

Dominance means that if alternative A_i is not inferior to alternative A_j in terms of all criteria, then combined with all criteria A_i is also not inferior to A_j and if with respect to at least one criterion A_i is superior to A_j, then A_i is superior to A_j combined with all criteria. For example, given a decision matrix as Table 3 since alternative A_1 is superior to A_2 with respect to every criterion aggregation rule satisfiying *Dominance* should give $A_1 \succ A_2$.

Criterion 2: Independence

Given two alternatives A_i and A_j, the preference of the two alternatives does not depend on the existence of other alternatives.

Independence means that adding or deleting an alternative would not change the rankings of the other alternatives. Aggregation rules satisfying *Independence* would not cause RR.

Criterion 3: Symmetric dominance

Given two alternatives A_i and A_j if criterion sequence $C_1, ..., C_m$ can be resorted as $C'_1, ..., C'_x, C'_{x+1}, ..., C'_{2x}, C'_{2x+1}, ..., C'_m$ such that

$$\frac{a_{ik}}{a_{jk}} = \frac{a_{j,x+k}}{a_{i,x+k}} > 1, \quad k = 1, 2, ..., x \ (x \leq m/2); \tag{2.1}$$

$$a_{ik} = a_{jk}, \quad k = 2x + 1, ..., m; \tag{2.2}$$

$$c'_k \geq c'_{x+k}, \quad k = 1, 2, ..., x; \tag{2.3}$$

then $A_i \geq A_j$. Furthermore only if $c'_k = c'_{x+k}$ ($k = 1, ..., x$), $A_i = A_j$, otherwise $A_i \succ A_j$.

Symmetric dominance means when the relavtive performances of two alternatives in terms of all the criteria are strictly symmetric, the preference of the two alternatives is determined by the contributions of each criterion to decision goal. For example, suppose a decision matrix is shown as Table 4. According to the definition of *Symmetric dominance*, rearrange Table 4 into Table 5 and 6 respectively. Aggregation rule satisfying *Symmetric dominance* should give $A_1 \succ A_2$ and $A_2 = A_3$.

Table 4. Example 3

Alternative	Criteria			
	C_1	C_2	C_3	C_4
	(0.4	0.2	0.2	0.2)
A_1	6	2	3	1
A_2	3	2	6	1
A_3	3	2	1	6

Table 5. Rearrangement 1 of example 3

Alternative	Criteria			
	C_1'	C_2'	C_3'	C_4'
	(0.4	0.2	0.2	0.2)
A_1	6	3	2	1
A_2	3	6	2	1

Table 6. Rearrangement 2 of example 3

Alternative	Criteria			
	C_1'	C_2'	C_3'	C_4'
	(0.2	0.2	0.4	0.2)
A_2	6	1	3	2
A_3	1	6	3	2

Criterion 4: Transitivity

Given alternatives A_i, A_j and A_k, if $A_i \succ A_j$, $A_j \succ A_k$, then $A_i \succ A_k$.

Transitivity means that if alternative A_i is preferred to alternative A_j, which is in turn preferred to alternative A_k, then alternative A_i is preferred to alternative A_k.

3. Typical Aggregation Rules

Many aggregation rules have been proposed to derive alternative rankings from a given decision matrix in AHP. In this section we present three typical aggregation rules.

Table 7. Example 4

Alternative	Criteria		
	C_1	C_2	C_3
	(0.2	0.4	0.4)
A_1	8	2	2
A_2	4	1	4
A_3	4	8	1

Aggregation Rule 1: Saaty's aggregation rule (SAHP)

Given decision matrix as Table 1, SAHP calculates priority vector P by (3.1).

$$\begin{aligned} P &= (p_1, p_2, ..., p_n)^T \\ &= \begin{pmatrix} \frac{a_{11}}{\sum_{i=1}^n a_{i1}} & \frac{a_{12}}{\sum_{i=1}^n a_{i2}} & \cdots & \frac{a_{1m}}{\sum_{i=1}^n a_{im}} \\ \frac{a_{21}}{\sum_{i=1}^n a_{i1}} & \frac{a_{22}}{\sum_{i=1}^n a_{i2}} & \cdots & \frac{a_{2m}}{\sum_{i=1}^n a_{im}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{a_{n1}}{\sum_{i=1}^n a_{i1}} & \frac{a_{n2}}{\sum_{i=1}^n a_{i2}} & \cdots & \frac{a_{nm}}{\sum_{i=1}^n a_{im}} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{pmatrix} \\ &= (\sum_{j=1}^m c_j \frac{a_{1j}}{\sum_{i=1}^n a_{ij}}, \sum_{j=1}^m c_j \frac{a_{2j}}{\sum_{i=1}^n a_{ij}}, \cdots, \sum_{j=1}^m c_j \frac{a_{nj}}{\sum_{i=1}^n a_{ij}})^T \end{aligned} \tag{3.1}$$

Aggregation Rule 2: Multiplicative AHP (MAHP)

Given decision matrix as Table 1, MAHP derives priority vector by (3.2).

$$\begin{aligned} P &= (p_1, p_2, ..., p_n)^T \\ &= (\prod_{j=1}^m a_{1j}^{c_j}, \prod_{j=1}^m a_{2j}^{c_j}, \cdots, \prod_{j=1}^m a_{nj}^{c_j})^T \end{aligned} \tag{3.2}$$

Aggregation Rule 3: Triantaphyllou's aggregation rule (TAHP)

Instead of calculating a vector to denote the combined priorities of alternatives, Triantaphyllou suggested decomposing a decision-making problem into a set of sub-problems and combining the partial solutions of these sub-problems to form overall alternative rankings (Triantaphyllou, 2001). For example suppose a decision matrix is given as Table 7. According to TAHP, this decision-making problem is decomposed into 3 sub-problems described with Table 8-10 respectively. Using SAHP to resolve the three sub-problems yields three partial rankings, which in combination demonstrate that the overall rankings of the three alternatives are $A_3 \succ A_1 \succ A_2$.

The procedures of TAHP are formally stated as follows. Given a decision matrix

Table 8. Subproblem 1 of example 4

Alternative	Criteria			Combined priorities	Derived ranking
	C_1	C_2	C_3		
	(0.2	0.4	0.4)		
A_1	8/12	2/3	2/6	0.5333	$A_1 \succ A_2$
A_2	4/12	1/3	4/6	0.4667	

Table 9. Subproblem 2 of example 4

Alternative	Criteria			Combined priorities	Derived ranking
	C_1	C_2	C_3		
	(0.2	0.4	0.4)		
A_1	8/12	2/10	2/3	0.4800	$A_3 \succ A_1$
A_3	4/12	8/10	1/3	0.5200	

Table 10. Subproblem 3 of example 4

Alternative	Criteria			Combined priorities	Derived ranking
	C_1	C_2	C_3		
	(0.2	0.4	0.4)		
A_2	4/8	1/9	4/5	0.4644	$A_3 \succ A_2$
A_3	4/8	8/9	1/5	0.5356	

shown as Table 1, build matrix $B = (b_{ij})_{n\times n}$, in which

$$b_{ij} = \frac{\sum_{k=1}^{m} c_k \frac{a_{ik}}{a_{ik}+a_{jk}}}{\sum_{k=1}^{m} c_k \frac{a_{jk}}{a_{ik}+a_{jk}}}, \quad i, j = 1, 2, ..., n; \tag{3.3}$$

If $b_{ij} > 1$, $A_i \succ A_j$ and $b_{ij} = 1$ gives $A_i = A_j$. Combining all the partial rankings yields overall alternative rankings.

4. Evaluation of Aggregation Rules against Evaluative Criteria

Evaluating the aggregation rules against the proposed criteria yields the following theorems.

Theorem 4.1. MAHP satisfies *Dominance*, *Independence*, *Symmetric dominance* and *Transitivity*.

Proof. Given decision matrix as Table 1, use (3.2) to calculate priority vector P.

(i) *Dominance* evaluation
For any alternative pair A_i and A_j, if with respect to every criterion C'_k $(k = 1, 2, ..., m)$, $a_{ik} \geq a_{jk}$, obviously

$$p_i = \prod_{k=1}^{m} a_{ik}^{c_k} \geq \prod_{k=1}^{m} a_{jk}^{c_k} = p_j \tag{4.4}$$

which means $A_i \geq A_j$. If and only if $a_{ik} = a_{jk}$ $(k = 1, 2, ..., m)$, $A_i = A_j$. Therefore MAHP satisfies *Dominance*.

(ii) *Independence* evaluation
Since adding or deleting alternative does not affect the calculation of other alternatives' priorities, RR does not occur using MAHP which means MAHP satisfies *Independence*.

(iii) *Symmetric dominance* evaluation
According to the definition of *Symmetric dominance*, given two alternatives A_i and A_j

$$\begin{aligned} p_i/p_j &= \frac{\prod_{k=1}^{x} a_{ik}^{c'_k} \prod_{k=x+1}^{2x} a_{ik}^{c'_k}}{\prod_{k=1}^{x} a_{jk}^{c'_k} \prod_{k=x+1}^{2x} a_{jk}^{c'_k}} \\ &= \prod_{k=1}^{x} (\frac{a_{ik}}{a_{jk}})^{c'_k} (\frac{a_{i,x+k}}{a_{j,x+k}})^{c'_{x+k}} \\ &= \prod_{k=1}^{x} (\frac{a_{ik}}{a_{jk}})^{c'_k - c'_{x+k}} \geq 1 \end{aligned} \tag{4.5}$$

which means $A_i \geq A_j$. If and only if $c'_k = c'_{x+k}$ $(k = 1, 2, ..., x)$, $A_i = A_j$. Therefore MAHP satisfies *Symmetric dominance*.

(iv) *Transitivity* evaluation
If $A_i \succ A_j$ and $A_j \succ A_k$,then $p_i > p_j > p_k$. Therefore $A_i \succ A_k$, which means MAHP satisfies *Transitivity*. □

Theorem 4.2. SAHP satisfies *Dominance* and *Transitivity*, but not *Independence* and *Symmetric dominance*.

Proof. Given decision matrix as Table 1, use (3.1) to calculate priority vector P. Suppose there are two alternatives A_i and A_j.

(i) *Dominance* evaluation
If $a_{ik} \geq a_{jk}$ with respect to every criterion C_k $(k = 1, 2, ..., m)$, obviously

$$p_i = \sum_{k=1}^{m} \frac{c_k a_{ik}}{\sum_{q=1}^{n} a_{qk}} \geq \sum_{k=1}^{m} \frac{c_k a_{jk}}{\sum_{q=1}^{n} a_{qk}} = p_j \tag{4.6}$$

which means $A_i \geq A_j$. If and only if $a_{ik} = a_{jk}$ $(k = 1, 2, ..., m)$, $A_i = A_j$, which means SAHP satisfies *Dominance*.

Table 11. Example 5

Alternative	Criteria		
	C_1	C_2	C_3
	(0.4	0.2	0.4)
A_1	2	1	1
A_2	1	2	1
A_3	8	1	8

Table 12. Decision matrix after deleting A_3 from example 5

Alternative	Criteria		
	C_1	C_2	C_3
	(0.4	0.2	0.4)
A_1	2	1	1
A_2	1	2	1

(ii) *Independence* and *Symmetric dominance* evaluation
Suppose a decision matrix is described with Table 11. According to *Symmetric dominance*, $A_1 \succ A_2$. However using SAHP to calculate priority vector yields

$$P = (0.1627, 0.1764, 0.6609)^T \tag{4.7}$$

which means $A_3 \succ A_2 \succ A_1$.

Eliminating A_3 from example 5 (see Table 11) yields new decision matrix shown as Table 12. Using SAHP to re-calculate the priority vector yields

$$P = (0.5333, 0.4667)^T \tag{4.8}$$

which means $A_1 \succ A_2$. Rank reversal occurs. Thus SAHP does not satisfy *Independence* and *Symmetric dominance*.

(iii) *Transitivity* evaluation
If $A_i \succ A_j$ and $A_j \succ A_k$, then $p_i > p_j > p_k$, which means $A_i \succ A_k$. Therefore SAHP satisfies *Transitivity*. □

Theorem 4.3. TAHP satisfies *Dominance*, *Independence* and *Symmetric dominance*, but not *Transitivity*.

Proof. Given decision matrix as Table 1 and alternative pair A_i and A_j, using

Table 13. Example 6

Alternative	Criteria		
	C_1	C_2	C_3
	(0.6	0.3	0.1)
A_1	8	2	1
A_2	8	1	8
A_3	4	8	2

TAHP to obtain the rankings yields

$$p_i = \sum_{k=1}^{m} \frac{c_k a_{ik}}{a_{ik} + a_{jk}}, \quad p_j = \sum_{k=1}^{m} \frac{c_k a_{jk}}{a_{ik} + a_{jk}} \tag{4.9}$$

(i) *Dominance* evaluation
If with respect to every criterion C_k $(k = 1, 2, ..., m)$, $a_{ik} \geq a_{jk}$, then obviously

$$p_i - p_j = \sum_{k=1}^{m} \frac{c_k(a_{ik} - a_{jk})}{a_{ik} + a_{jk}} \geq 0 \tag{4.10}$$

which means $A_i \geq A_j$. If and only if $a_{ik} = a_{jk}$ $(k = 1, 2, ..., m)$, $A_i = A_j$. Therefore TAHP satisfies *Dominance*.

(ii) *Independence* evaluation
Since adding or deleting alternative does not affect the rankings of other alternative pair, TAHP satisfies *Independence*.

(iii) *Symmetric dominance* evaluation
According to the definition of *Symmetric dominance*, set

$$s_k = \frac{a_{ik}}{a_{jk}} = \frac{a_{j,x+k}}{a_{i,x+k}} > 1, \quad k = 1, ..., x; \tag{4.11}$$

$$\begin{aligned} p_i - p_j &= \sum_{k=1}^{x} \frac{c'_k(a_{ik} - a_{jk})}{a_{ik} + a_{jk}} + \sum_{k=x+1}^{2x} \frac{c'_k(a_{ik} - a_{jk})}{a_{ik} + a_{jk}} \\ &= \sum_{k=1}^{x} \frac{c'_k(a_{ik} - a_{jk})}{a_{ik} + a_{jk}} + \sum_{k=1}^{x} \frac{c'_{x+k}(a_{i,x+k} - a_{j,x+k})}{a_{i,x+k} + a_{j,x+k}} \\ &= \sum_{k=1}^{x} \frac{s_k - 1}{s_k + 1}(c'_k - c'_{x+k}) \geq 0 \end{aligned} \tag{4.12}$$

If and only if $c'_k = c'_{x+k}$ $(k = 1, 2, ..., x)$, $A_i = A_j$, which means TAHP satisfies *Symmetric dominance*.

(iv) *Transitivity* evaluation
Suppose a decision matrix is shown as Table 13. Decompose Table 13 into three sub-problems shown as Table 14-16. Since $A_1 \succ A_2$, $A_2 \succ A_3$ and $A_3 \succ A_1$, TAHP does not satisfy *Transitivity*.

Table 14. Subproblem 1 of example 6

Alternative	Criteria			Combined priorities	Derived ranking
	C_1	C_2	C_3		
	(0.6	0.3	0.1)		
A_1	8/16	2/3	1/9	0.5111	$A_1 \succ A_2$
A_2	8/16	1/3	8/9	0.4889	

Table 15. Subproblem 2 of example 6

Alternative	Criteria			Combined priorities	Derived ranking
	C_1	C_2	C_3		
	(0.6	0.3	0.1)		
A_2	8/12	1/9	8/10	0.5133	$A_2 \succ A_3$
A_3	4/12	8/9	2/10	0.4867	

Table 16. Subproblem 3 of example 6

Alternative	Criteria			Combined priorities	Derived ranking
	C_1	C_2	C_3		
	(0.6	0.3	0.1)		
A_1	8/12	2/10	1/3	0.4933	$A_3 \succ A_1$
A_3	4/12	8/10	2/3	0.5067	

□

In this section, we have evaluated three typical aggregation rules against the evaluative criteria proposed in Sec. 2. Though the results show that only MAHP satisfies all of the four criteria, we think it is still very early to declare that other aggregation rules are invalid because it is still arguable whether all of the four criteria are indispensible to a valid aggregation rule. For example in number theory *Transitivity* is considered as an axiom which means given rationals x, y and z, if $x > y$ and $y > z$, then certainly $x > z$. However in decision-making field *Transitivity* does not always hold. For example if we are required to predict the mutual competition results of three football teams, A, B and C, then it makes sense if we give rankings $A \succ B$, $B \succ C$ and $C \succ A$.

5. Conclusions

In this paper we proposed four criteria to evaluate the validity of AHP aggregation rules. Three typical AHP aggregation rules, namely SAHP, MAHP and TAHP, are evaluated against the criteria and the results show that only MAHP satisfies all of the four criteria. However as it is still arguable that whether all of the four criteria should be deemed as indispensible to a valid aggregation rule, we think it is still too early to declare that aggregation rules not satisfying all of the four proposed criteria are invalid and we expect more research on this issue.

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