AHP/ANP– Issues and Insights for the future

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Link to Talk on YouTube
https://youtu.be/t-5WDIJXaug

The International Symposiums

#	Year	Location	Venue	Chair	Papers
1	1988	Tianjin, China	Tianjin University	Shubo Xu	87
2	1991	Pittsburgh, United States	University of Pittsburgh	Luis Vargas	42
3	1994	Washington, D.C., United States	George Washington University	Ernest Forman	52
4	1996	Vancouver, Canada	Simon Fraser University	William Wedley	81
5	1999	Kobe, Japan	Unknown	Eizo Kinoshita	79
6	2001	Berne, Switzerland	Hotel Allegro	Klaus Dellmann	57
7	2003	Nusa Dua, Indonesia	Melia Bali Hotel	Kirti Peniwati	74
8	2005	Honolulu, United States	University of Hawaii	Jason Levy	75
9	2007	Viña del Mar, Chile	Hotel del Mar	Claudio Garuti	66
10	2009	Pittsburgh	University of Pittsburgh	Thomas Saaty	85
11	2011	Sorrento, Italy	Hilton Sorrento Palace	Emilio Esposito	162
12	2013	Kuala Lumpur, Malaysia	Istana Hotel	Rafikul Islam	92
13	2014	Washington, United States	Grant Hyatt Hotel	Enrique Mu	185
14	2016	London, UK	Hilton Paddington	Leandro Pecchia	
15	2018	Hong Kong	Grand Hyatt Hotel	Luis Vargas and Jennifer Shang	90
16	2020	World Wide Web		Enrique Mu, Antonella Petrillo and Elena Rokou	

Bill's Background

- **Born and raised in North Vancouver, Canada**
- **Undergraduate education: University of British Columbia. B Comm**
- **Graduate education: Columbia University, New York, MBA and PhD.**
- **Before receiving PhD, 3 years at the University of Libya, Benghazi.**
- **Returned to Canada, Simon Fraser University. My specialties were operations management, international business and decision making.**
- **I have always been interested in how to incorporate qualitative factors into quantitative decision models. In 1980, while in Australia, that interest lead me to AHP.**
- **I programmed the eigenvector routine and found it to be uncanny in the ability to capture preferences. I was hooked!**
- **I first met Tom and Rozann Saaty when they came to Simon Fraser to give a seminar about 1983.**

AHP/ANP: Magic or Black Box

- **Much of AHP/ANP is like magic – it is easy to use and intuitive in capturing peoples' preferences. It has a way of fascinating all who see it perform.**
- **When Orrin Cooper (2017) used "magic" to describe AHP/ANP, I said I was uneasy with that word because it implied deception and slight of hand. We would not want that.**
- **Later, upon reflection, I decided that there is a positive magical component to AHP/ANP. It is just that the models get complex and are like a black box to many.**
- **We need to look inside the "black box" to understand what is going on.**

Functional inconsistency!

- **While being an ardent advocate for AHP, my research has been shaped by various people who challenged my thoughts.**
- **Those challenges caused inconsistency in my mind and led me in different directions.**
- **As Tom Saaty frequently said, some inconsistency is functional. It provides the impetus for investigation, innovation and advancements .**
- **I will tell you about some of the people who created functional inconsistencies in my mind.**
- **I will also present some of the questions that arose from these functional inconsistencies. They are the themes of the remainder of my presentation.**

Overview of questions behind themes

1. **Do we need to know natural zero to do comparisons?**
2. **Do we need to do all comparisons?**
3. **Do AHP priorities have a unit?**
4. **How do you get the full benefit of ratio intensities?**
5. **Are criteria weights dependent upon alternatives?**
6. **Can criteria weights be independent of alternatives?**
7. **Can derived scales be used as scales for measurement?**
8. **Do benefits and cost priorities have to be commensurate?**
9. **Can a unit interpretation be applied to the supermatrix?**

Background

- Central to all the previous themes is the following quotation
- "And therein lie **both the advantage and dilemma** of AHP [and ANP]. We do not need explicit knowledge of the underlying unit of measure to derive a ratio scale, yet the derived scale has a unit." (Wedley & Choo, 2011; emphasis added)"
- Advantage: we can get scales from measurements (comparisons) without having any prior standards. Although each comparison has a unit, we do not know the ultimate unit of the scale that evolves from the comparisons
- Dilemma: sum to unity priorities have an obscure unit. As we use the priorities, too frequently it is without recognition of the unit. Accordingly, we often aggregate units incorrectly.
- The AHP/ANP community has not paid enough attention to the units of their scales.

Themes –Advantages and Dilemmas

- Themes 1 & 2 highlight the advantages of AHP/ANP scales
- Themes 3 describes the nature of AHP/ANP scales
- Themes 4-7 describes how they can be used in both and advantageous and deleterious manner.
- Theme 9 asserts that the unit is also important in Supermatrices.

Theme 1: Natural zero and making comparisons

- **1998, plenary speaker alongside with Jonathan Barzilai, International Conference on Multi-Objective Programming in Quebec City.**
- **Jonathan, a very competent mathematician, is one of the most strident critics of AHP. The presentation was set up like a debate.**
- **Jonathan presented a picture similar to the following. He then asked participants to compare the buildings according to height.**

Requirements for Comparisons

- Absolute zero is the origin for all ratio intensities, but it has no impact upon ratios. A ratio is one intensity (something) divided by another intensity (something). Something divided by zero (nothing) is impossible. Division by zero is undefined.
- To do comparisons, we must have the ability to sense the intensities of the property being evaluated.
- We need at least two objects with a degree of intensity so that one of them can be the unit for comparison.
- We also need reasonable knowledge and expertise to make accurate comparisons

Accuracy and Consistency

- Bernasconi M., Choirat, C. and Seri, R. (2010) The Analytic Hierarchy Process and Theory of Measurement, *Management Science*, 56, 4, 699-711.

Table 1 Items Compared in the Three Experiments

Distances from Milan (km)	Games of chance (probability)	Rainfall in November 2001 (mm)
1. Naples (658 km)	Take 1 heart out of a pack of 52 cards (1/4)	Prague (26 mm)
2. Venice (247 km)	Take 1 ace out of a pack of 52 cards (1/13)	Athens (53 mm)
3. Rome (491 km)	Get at least 3 in a roll of a 6-sided dice (4/6)	Copenhagen (48 mm)
4. Turin (124 km)	Get 6 in a roll of a 6-sided dice (1/6)	London (54 mm)
5. Palermo (885 km)	Get heads in a toss of a coin (1/2)	Rome (127 mm)

Sources. http://www.chemical-ecology.net/java/lat-long.htm for distances, Fremy and Fremy (2001) for rainfall.

- Consistency was good in all three experiments. Accuracy was abysmal for estimating rainfall
- Thus, it possible to be consistent while at the same time being inaccurate.

Conclusions re making comparisons

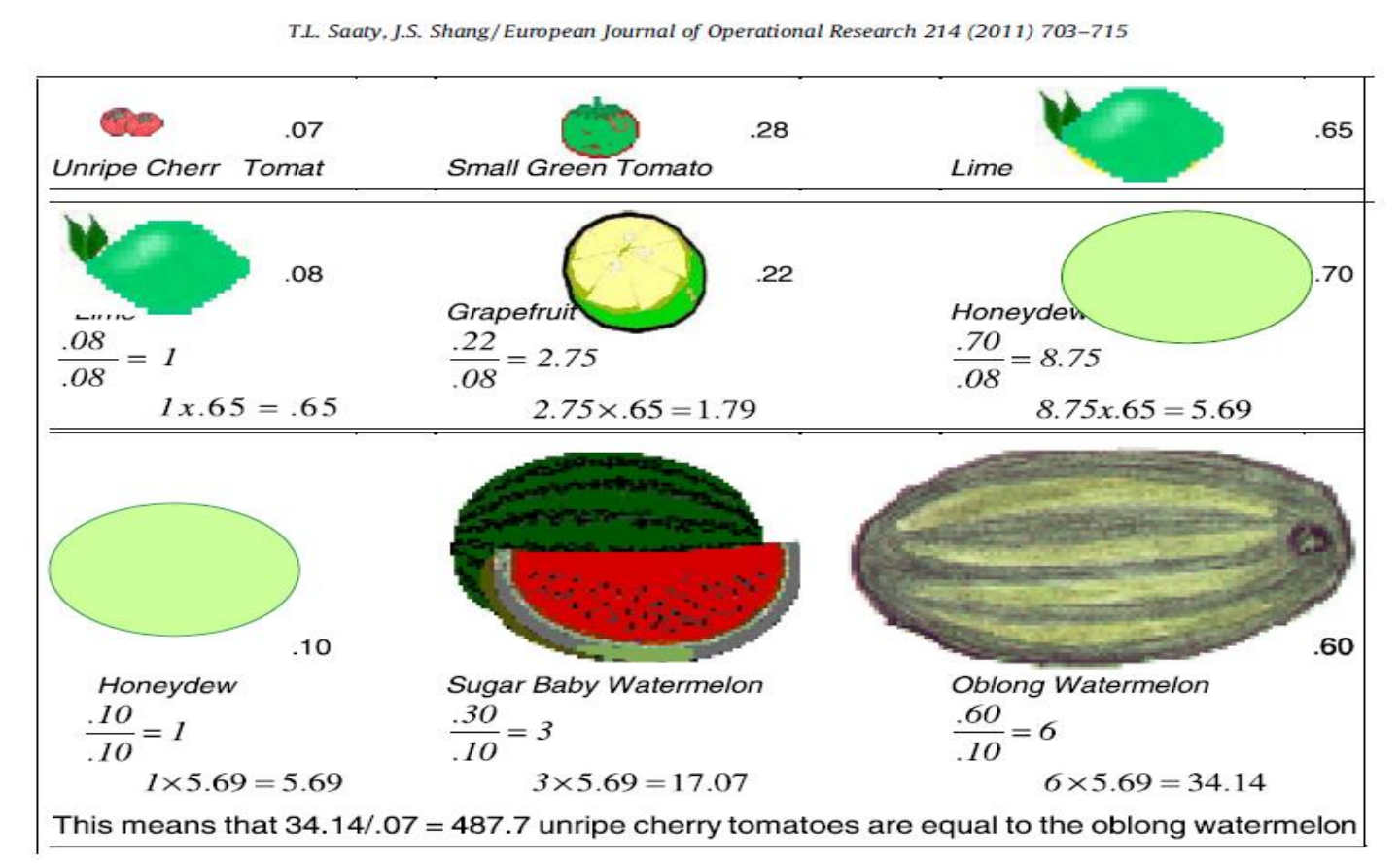
- Although unwritten, the basic assumption of AHP/ANP is that there is an underlying ratio scale that we are trying to estimate.
- We seldom know this scale directly but we want our pairwise ratio matrix (**PRM**) to be accurate so that our eigenvector will also be accurate.
- The entries to the comparison matrix are ratio values. This includes values from the fundamental scale.
- Some thing (an intensity) divided by no thing (zero) does not produce a ratio. It must be the ratio of two intensities.
- It is the ratio feature that distinguishes AHP matrices from other types of comparison matrices. Thus it is advisable to call it the Pairwise Ratio Matrix (**PRM**).
- **Remember, a Low Consistency Ratio ≠ Accuracy.**
- Consistency is desirable, but more important is knowledgeable participants who make the comparisons.

Theme 2: Do we need all comparisons?

- **Users complain about an excessive number of comparisons. I sympathize with them.**
- **Only n-1 comparisons are required to determine a priority vector. Extra comparisons are useful for the consistency index, but there is evidence that each additional comparison yields diminishing benefits. Accordingly, some redundancy is useful, but a reduction in the number of comparisons makes sense.**
- **One of my areas of research has been on incomplete matrices, how to select the first n-1 comparisons and how to predict the consistency ratio.**
- **Many researchers have contributed to this problem with good ideas. Nonetheless, it does not seem that comparison reduction has been introduced into software.**

Theme 3: Do AHP priorities have a unit?

To illustrate the answer to this question, I will use data from the well known problem of linking sizes of fruits across several magnitudes.



Intensity of one object

- **Consider a single object (alternative) that possesses a property (criterion) that interests us (the size of various fruits).**
- **Intensity is the magnitude or degree of the property that the object possesses. The degree of intensity can be visualized as a line distance from natural zero.**
- **Object A: 0——a**  **(unripe cherry tomato)**
- **Natural zero, the origin, signifies an object that has absolutely no intensity (i.e. does not possess the property). It is a true absolute zero that represents "nothingness".**
- **To make paired comparisons, we do not need to identify, locate or specify natural zero. We just have to be able to sense the intensity the object is away from nothingness.**

Ratios with the intensity of a second object

- **Next, consider a second object that possesses greater intensity of the same property.**
- **Object A: 0——a**  **(unripe cherry tomato)**
- **Object B: 0————————————b**  **(small green tomato)**
- **With a second intensity, we can make a paired comparisons that are in ratio form.**
  - **Intensity B/Intensity A= 4/1 (A is the unit, scale is [1,4]**
  - **Intensity A/Intensity B= 1/4 (B is the unit, scale is [0.25, 1]**
- **Expressed as priorities that sum to unity, the scale for this comparison is [0.20, 0.80]. This scale is still ratio, since B/A=4 and A/B=1/4. However, the unit is obscure.**
- **Such sum to unity scales are called a relative ratio scales. We must ask ourselves – what are they relative to?**

Definition of a ratio scale

- **Ratios are one thing compared to another thing -- intensity of one object to the intensity of another object. When both things relate to the same property, we can get a ratio scale for that property.**

- *The ratio type takes its name from the fact that measurement is the estimation of the ratio between a magnitude of a continuous quantity and a unit magnitude of the same kind (Michell, 1997, 1999)*

- **Properties of ratio scales:**
  - **Origin is absolute zero**
  - **Multiplication by a positive constant ($b>0$, $b\neq1$) transforms the scale to new values with a different unit of measure. (i.e. a similarity transform just expresses the numbers in a different unit of measure).**
  - **Such a proportional transformation does not change the ratios between objects.**

- **With n objects of different intensities of the property, we get a measurement scale for those objects by comparing them to some object that has unit intensity. That unit object may be one of the n objects or some other object.**

The Basics: Ideal and Relative priorities

- **Object A:**
- **Object B:**

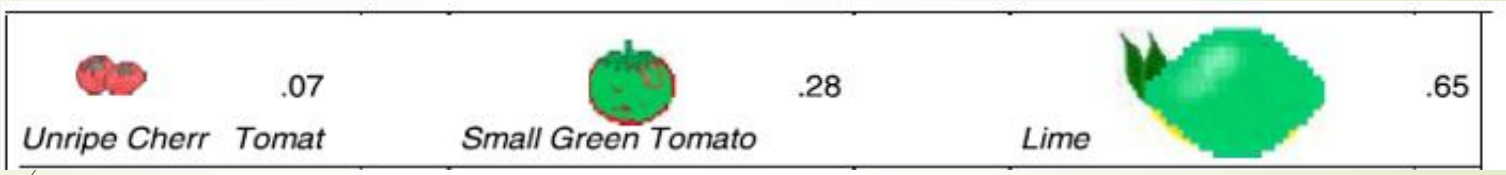
	Ideal priorities	Relative priorities
Unripe Cherry Tomato	0.25	0.20
Small Green Tomato	1	0.80
Unit =	1	1

Unit is an abstract object

- **For ideal priorities, it is clear that the more dominant object (small green tomato) is the unit.**
- **For relative priorities, the unit is more difficult to understand because the unit is not one of the objects. In the relative case, the unit is an abstract object that possesses the sum of intensities. The scale values are relative to that abstract object that represents all included intensities.**

The Basics: a third object

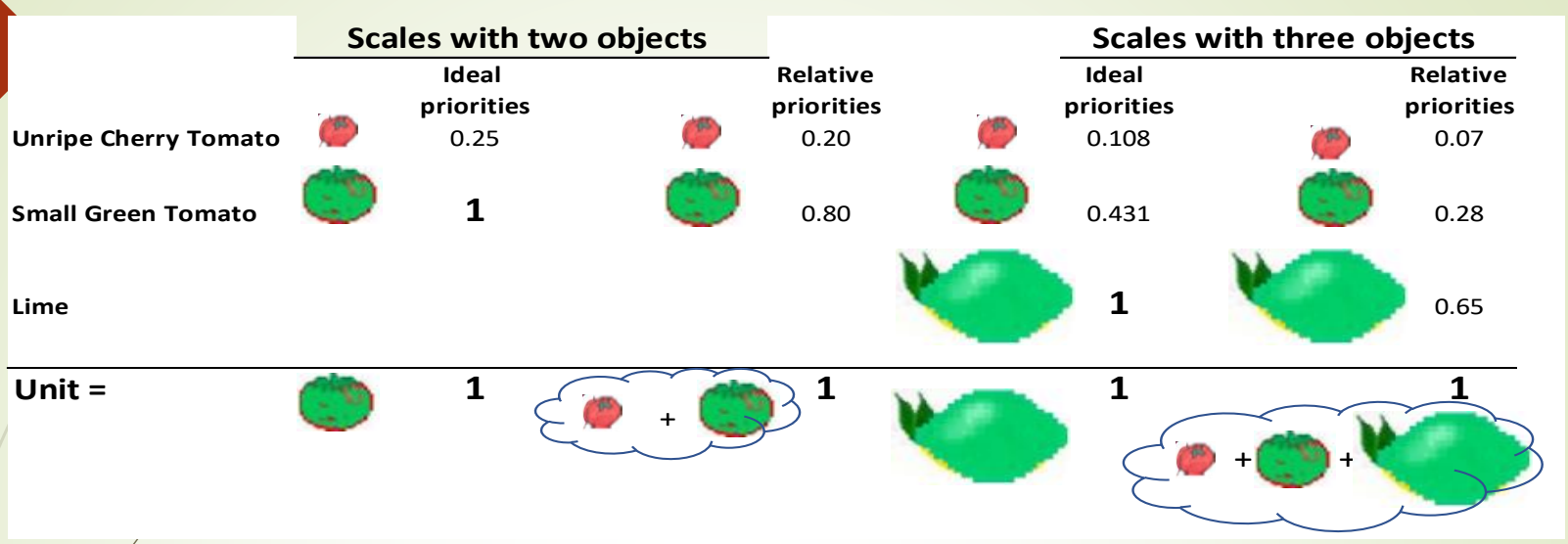
- **Object A: 0 ——— a**
- **Object B: 0 ——————————— b**
- **Object C: 0 ——————————————————————— c**



- **Object C (a Lime) is introduced. It is 9.3 times more intense than Object A and 2.3 times more intense than Object B.**

- **As shown above, relative priorities with three objects are different.**

Different units for different scales



- Notice that each scale has a different unit. Essentially, each scale is measuring the intensities with a different referent as the unit.

  "Although the sums are all equal to one, what is often ignored is that "one [1] *here is not necessarily equal to one* [1] *there". (Zahir, 2007)*

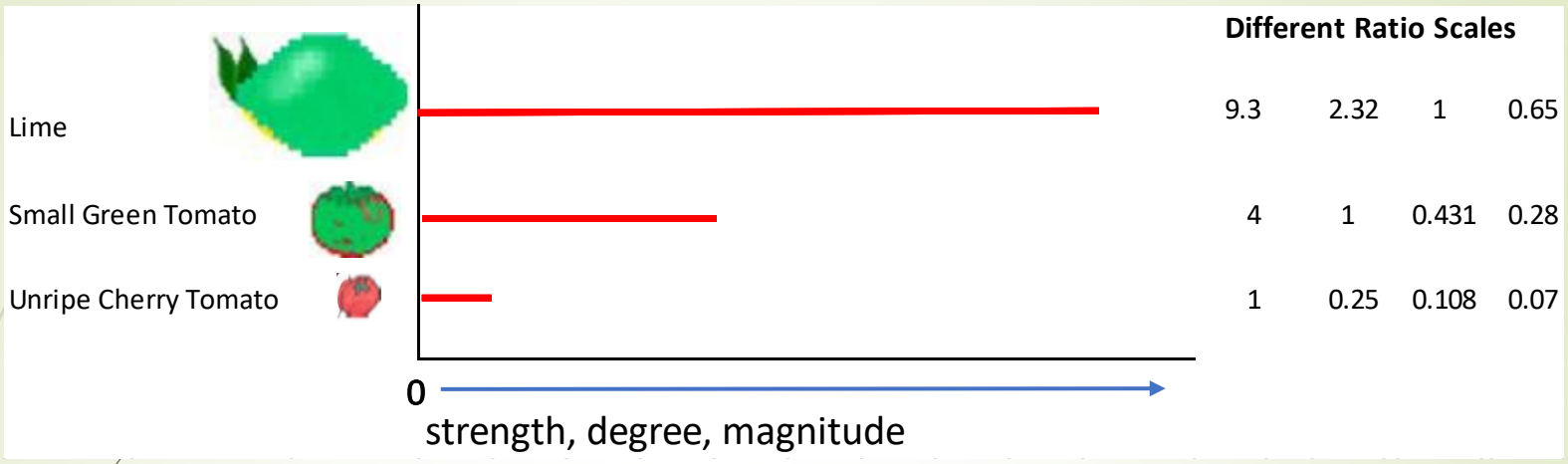
- Multiplying any of the above scales by a positive constant, ($b>0$, $b\neq1$) converts the scale to a different unit. We do such multiplication all the time in both AHP and ANP. When multiplying ratio scales by a positive constant, we should be aware that the units change.

Follow the Units

- A common saying for solving a commercial crime is to “Follow the Money”
- In AHP/ANP, there is both an advantage and dilemma associated with derived scales. The advantage is that we do not need explicit knowledge of the underlying unit to derive the scale. The dilemma is that once we get the scale it does have a unit that is obscure.
- When aggregating, things get complicated because we frequently change the unit of priority vectors when we multiply by a criterion weight or a rescaling factor.
- We should not ignore the unit nor take it for granted.
- To understand what is going on, it is useful to do two things.
  1. Recognize that a one [1] here is not necessarily equal to a one [1] there.
  2. Follow what happens to units.

Theme 4: Use the full intensity of ratios

Different numbers represent relative intensity

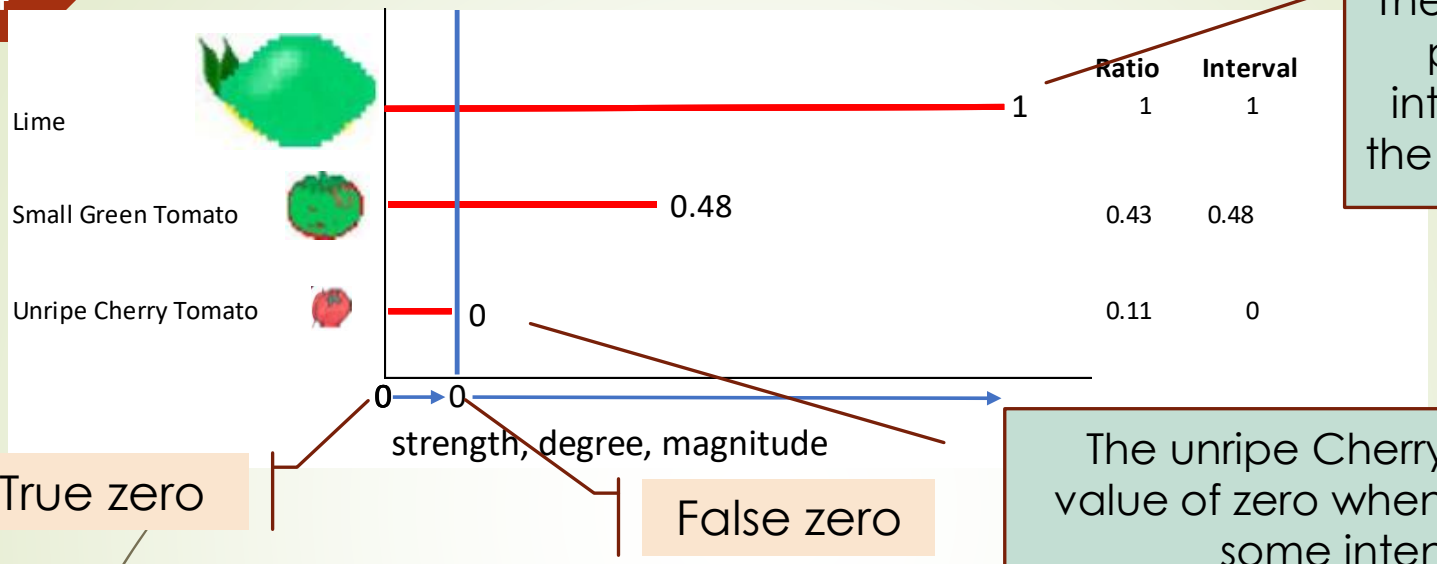


Remember that the ratio numbers are measures of "relative intensity" – relative to some unit that is arbitrary. Different units lead to different scales.

Numbers enable us to measure and have a scale.

No matter what numbers are used, intensities stay the same. So too do the ratios between any two objects.

Conversion to a MAUT/MAVT scale



MAUT/MAVT uses a partial interval of the intensity as the unit of measure when establishing criteria weights (i.e. they compare the interval units of the criteria).

Accordingly, they are very specific about the type of questions they ask when determining criteria weights.

Conversions to less powerful scales

- We can always convert a ratio scale to interval or ordinal scales.
- And for convenience, we can also use the ratio scales in an interval or ordinal manner.
- When we ignore the ratio property, we lose information and analytic power.
- There are many applications where we can improve with ratio measures.

Some examples

- Luis Vargas (2009, 2016) – voting with intensities.
- Enrique Mu (2017) – eyewitness identification
- Polling – What if we did not simply ask which candidate is preferred, but also by how many times.
- Corporate strategies. Often consensus is achieved for strategies that subsequently fail. Asking strength of conviction tells us whether people are lukewarm or strongly committed to a strategy.
- Military, governmental, and international and NGO decisions are also ripe for new ratio procedures.

Theme 5: Are criteria weights dependent upon alternatives?

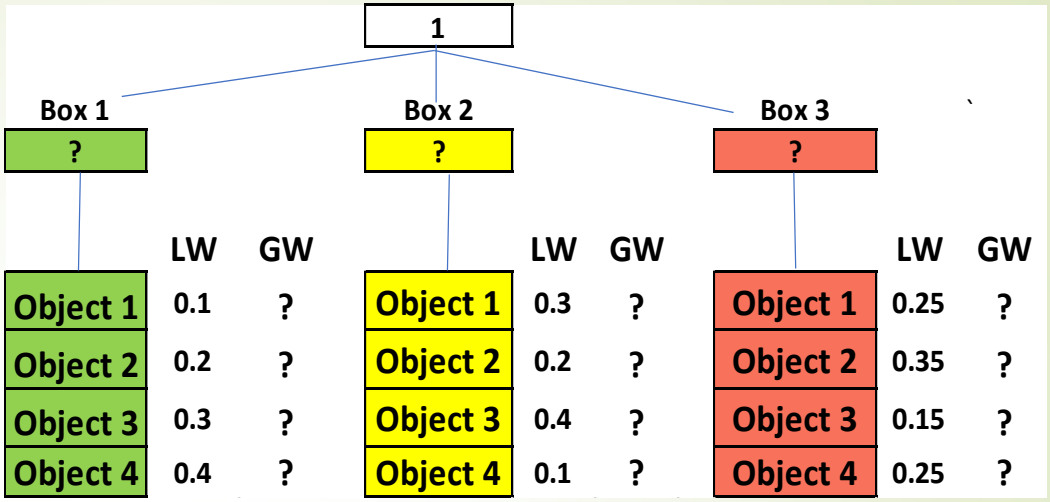
- My colleagues, Bert Schoner and Eng Choo joined my investigation of AHP techniques.
- We discussed functional inconsistencies that lead to productive joint research.
- Our main thrust was on the rank reversal problem that was illuminated by Belton and Gear.
- Our approach was to question how criteria weights are derived and whether global weights are commensurate before they are summed to get a composite result.

Bottom up approach

- A bottom up approach means you establish priorities for alternatives before you set the criteria weights that will transform them into global priorities.
- Ernest Forman, the developer of the AHP package Expert Choice use to say that you get a very different perspective if you approach the hierarchy from bottom to top instead of top to bottom.
- From a “bottom up” approach, you are confronted with the question: **What criteria weights create commensurate values across the various criteria**?

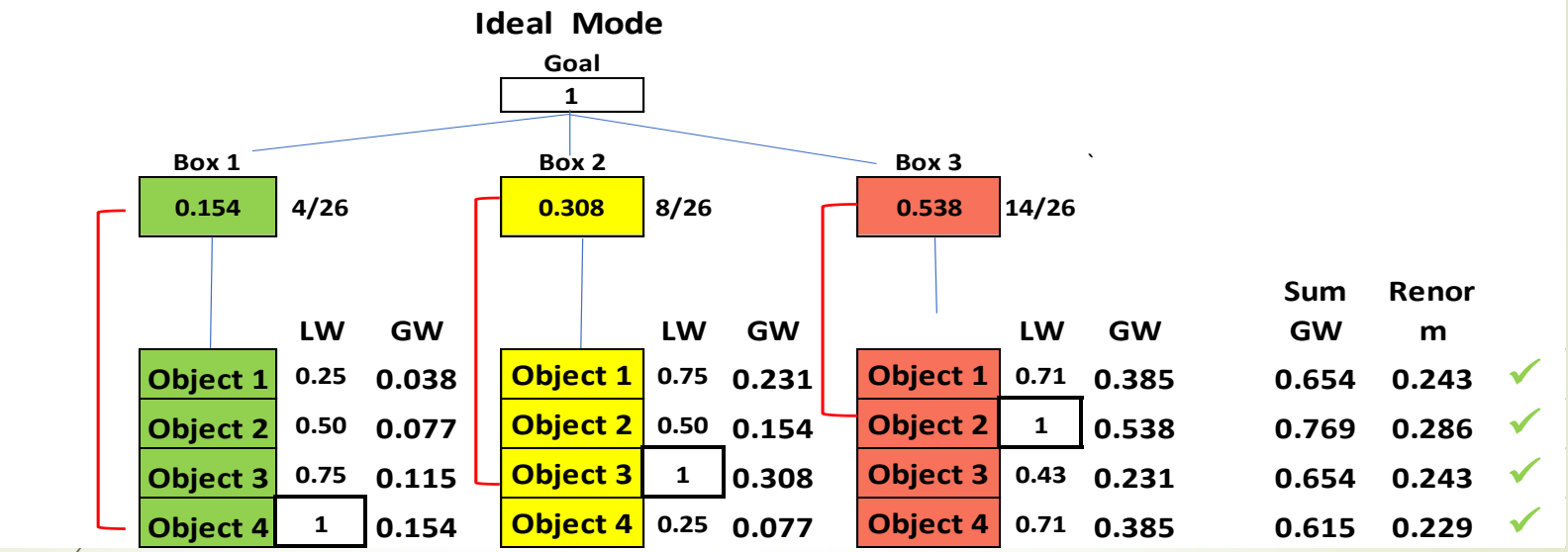
Example with 3 boxes with 4 objects (Vargas, 1997)

	Box 1	Box 2	Box 3	Total	True Weights
Object 1	1	6	10	17	0.243
Object 2	2	4	14	20	0.286
Object 3	3	8	6	17	0.243
Object 4	4	2	10	16	0.229
Total	10	20	40	70	1



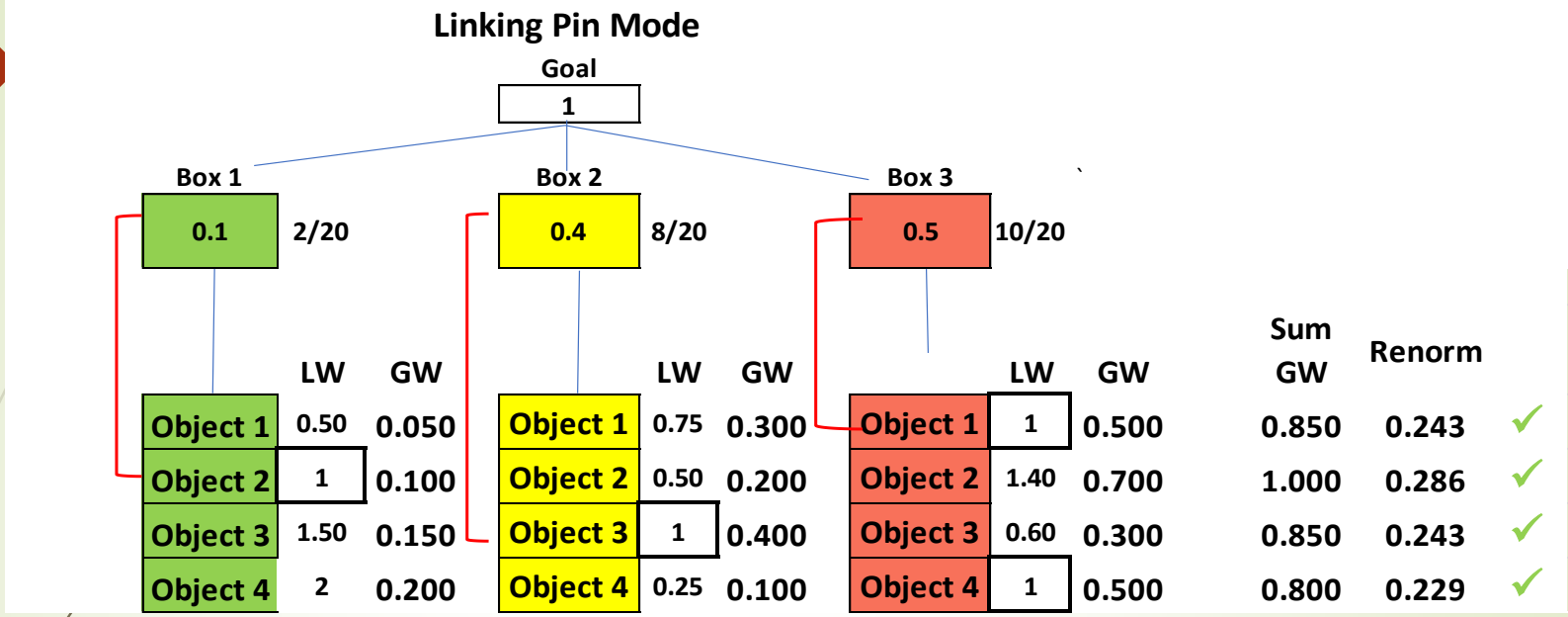
- Question: What cluster weights should be used to convert those local weights to a common unit representing the entire hierarchy.
- Each cluster sums to unity, but a one here is not equal to a one there.
- If we could get valid weights for the units of each cluster, we could transform local weights to global weights of one scale across all items.

Ideal mode solution



- Since the ideals are the units for the colored vectors, we need to find criteria weights for those units. If correctly done, the other objects in the box will get their correct weight via their relationship to the unit.
- For example, Object 2 is 0.5 of the unit in Box 1. It therefore receives 0.5 of the Box 1 criterion weight (0.5 * 0.154 = 0.077).
- This "bottom up" procedure was used by Belton and Gear in their original rank reversal article. **Note that the ideals are links between the three boxes**.

Linking Pin Solution



- Schoner, Wedley & Choo (1993) generalized Belton and Gear (1983) so that any object of the colored vectors could be the link. Shown above are Objects 2, 3 and 1 as the links. Red lines show the links
- The selected object is pinned to the higher level such that it is the link between levels and across clusters.
- Note again that this bottom up approach establishes criteria weights that are dependent upon the links that are chosen. The correct criteria weights link to get the correct composite ratios.

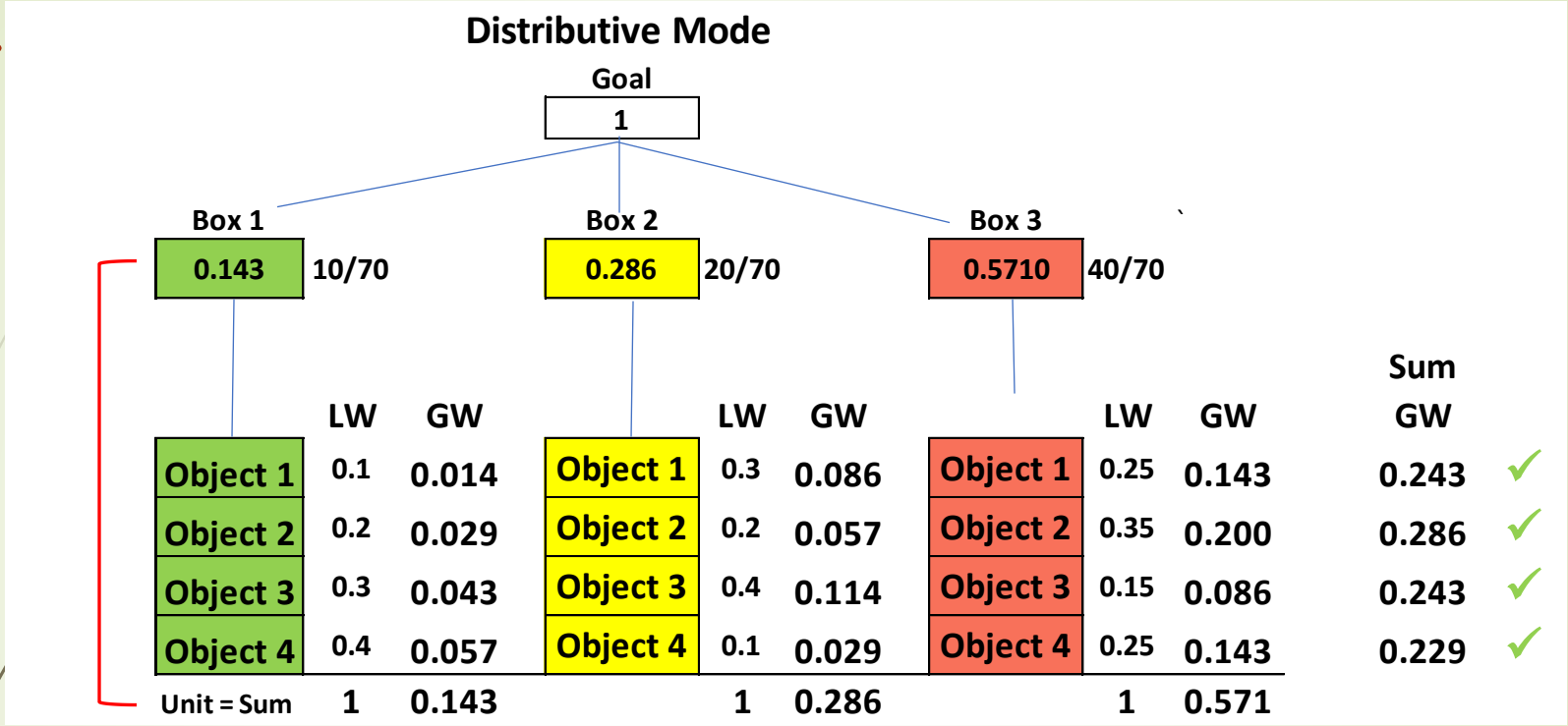
Linking pins and rank reversal

- Rank reversal is the change in rank of a composite result upon the addition or deletion of an alternative.
- Rank reversal does not occur when
  - There is only a single criterion
  - A linking pin mode is used and the links remain the same
- If the links remain the same, then calculated global weights of the original alternatives will remain unchanged. With no change in their measurement, there can be no change in rank.

Theme 6: Top Down – independence of criteria

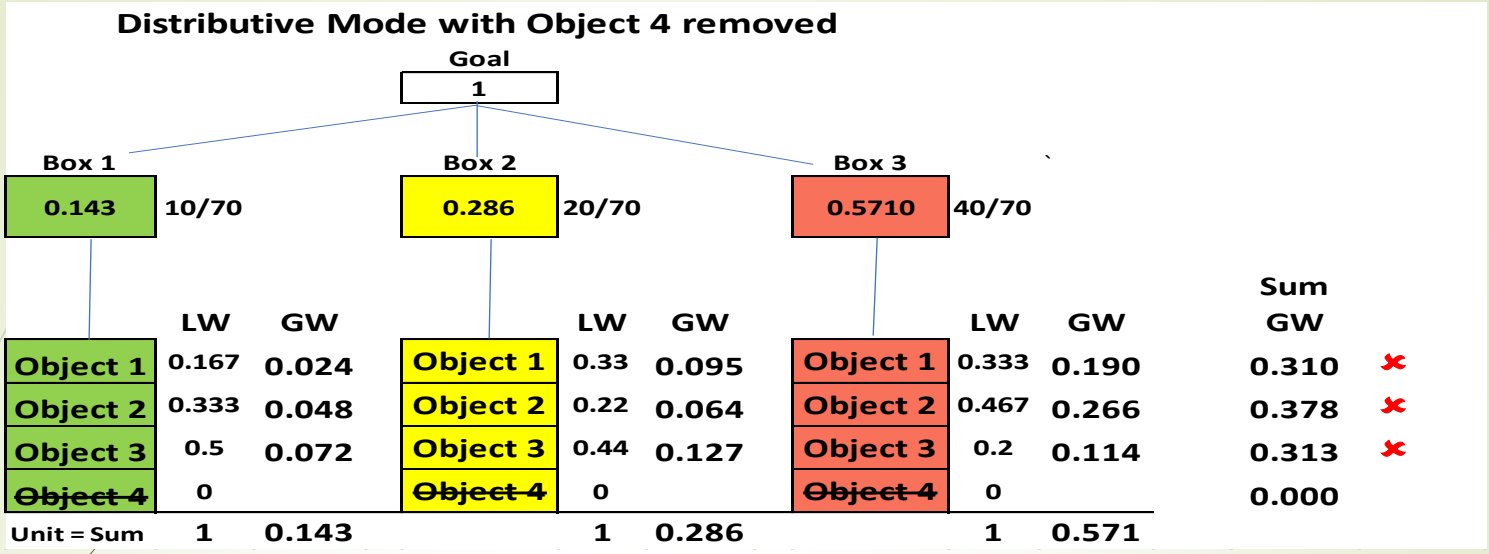
- The distributive mode is a “top down” procedure in which criteria weights are established first, independent of what alternatives are below.
- The establishment of criteria weights at the higher level establishes a homogeneous scale spanning across the hierarchy.
- Components of that homogenous scale are then distributed to whatever elements are below.
- If alternatives are added or deleted, the distribution below will be different.

Distributive Mode



- The unit for local weights that sum to unity is an abstract object representing an amalgamation of all objects. (remember the unit for fruit scales).
- Shown above are the criteria weights for those abstract objects. Using them, the correct composite priorities are produced.

Distributive mode, Object 4 removed



- We removed Object 4, renormalized local weights and distributed the criteria weights downward.
- Notice that local weights still sum to unity but take on different values. Different values for the same objects means the units of measure has changed. The renormalization (multiplication by a positive constant) produced a new unit of measure that is an abstract amalgamation of 3, not 4 objects.
- While this change did not cause rank reversal, the change in composite priorities did lead to incorrect ratios. Whereas Objects 1 and 3 should be equal, Object 3 now has a larger priority. As well the original ratios of Object 2 to other object has changed.
- Composite ratios change because renormalization produced different units that in turn produce different composite ratios. If the change in ratios is sufficient enough, rank reversal can occur.

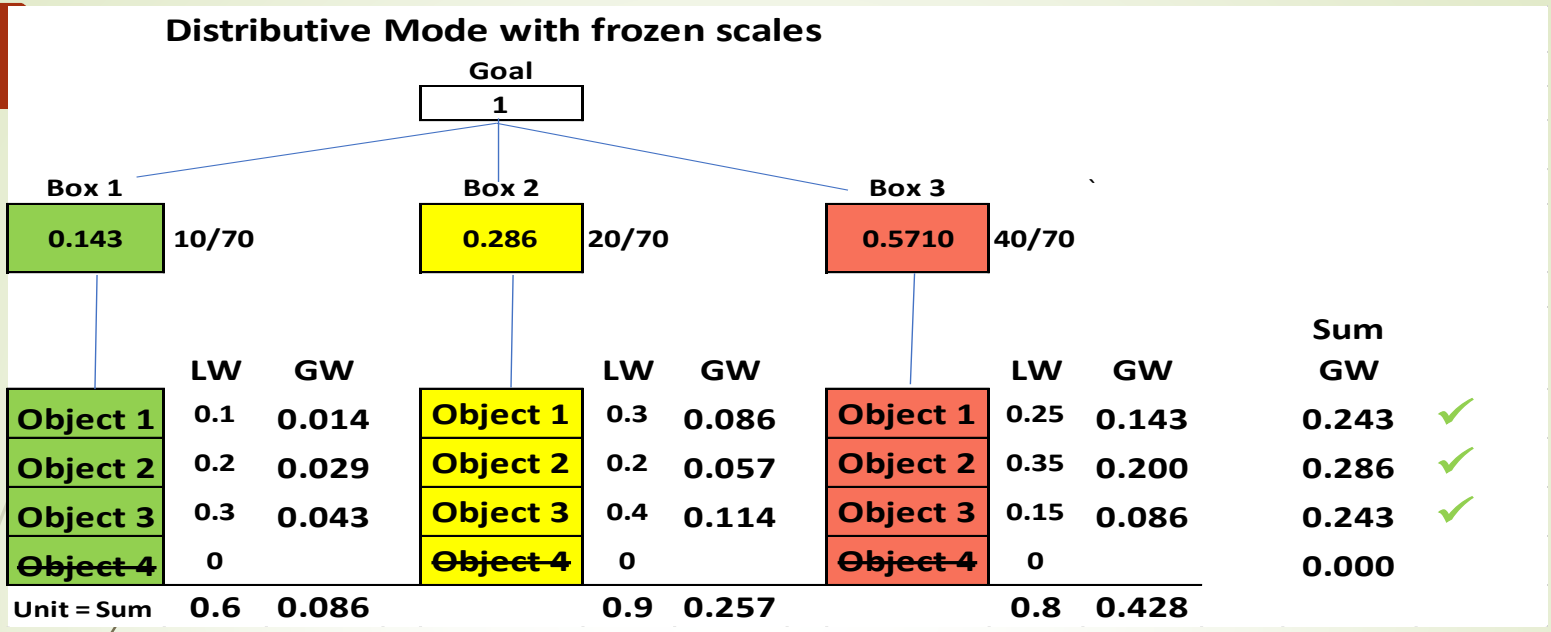
Do we have a problem?

	Box 1	Box 2	Box 3	Total	True Weights
Object 1	1	6	10	17	0.243
Object 2	2	4	14	20	0.286
Object 3	3	8	6	17	0.243
Object 4	4	2	10	16	0.229
Total	10	20	40	70	1

	Box 1	Box 2	Box 3	Total	True Weights	Sum GW
Object 1	1	6	10	17	0.315	0.310 ✘
Object 2	2	4	14	20	0.370	0.378 ✘
Object 3	3	8	6	17	0.315	0.313 ✘
Object 4				0		0.000
Total	6	18	30	54	1	

- With Object 4 removed, the intensities of the other objects are still the same. The true weights sum to a different 1, but Object 1 is still equal to Object 3 as it should be.
- But, the solution with priorities has Object 3 with a larger value than Object 1. There has been no rank reversal, but composite ratios have changed. Why?
- The problem is how ratios change, not that ranks have changed.
- In the rank reversal argument, we in the AHP/ANP community failed. We failed to analyze the problem on a ratio basis. We failed to use the full strength of intensities. We resorted to a lower order of measurement. We adopted the other side's criterion: rank.
- Had we analyzed the problem on a ratio basis, not ranks, I think we would have come to clearer understanding of what is happening. The change in ratios is associated with the change in units.

Object 4 removed, unchanged scales



- Here, Object 4 has been removed, but the local weights were not renormalized. With the units of measure unchanged, the same values and ratios are maintained.
- If we add objects to the pre-existing scales where units remain unchanged, those new objects take their values in the same pre-existing units, not different units. And with units unchanged, there will be no rank reversal.
- This idea implies that there can be a template hierarchy with fixed priorities. That is the idea behind benchmark measurement.

Theme 7: Scales used as Benchmarks

- Long ago (1996), Matti Verkasalo of Nokia complained that his executives were reluctant to use AHP because there were too many comparisons.
- They said it was repetitive, especially when the corporate criteria for each decision were the same.
- Matti requested some method to make decisions without repeating the comparisons each time.
- From that conversation grew the concept of benchmark measurement.
- Benchmark measurement is based upon fixed units of measure. Like the ratings method of AHP it can handle many alternatives.

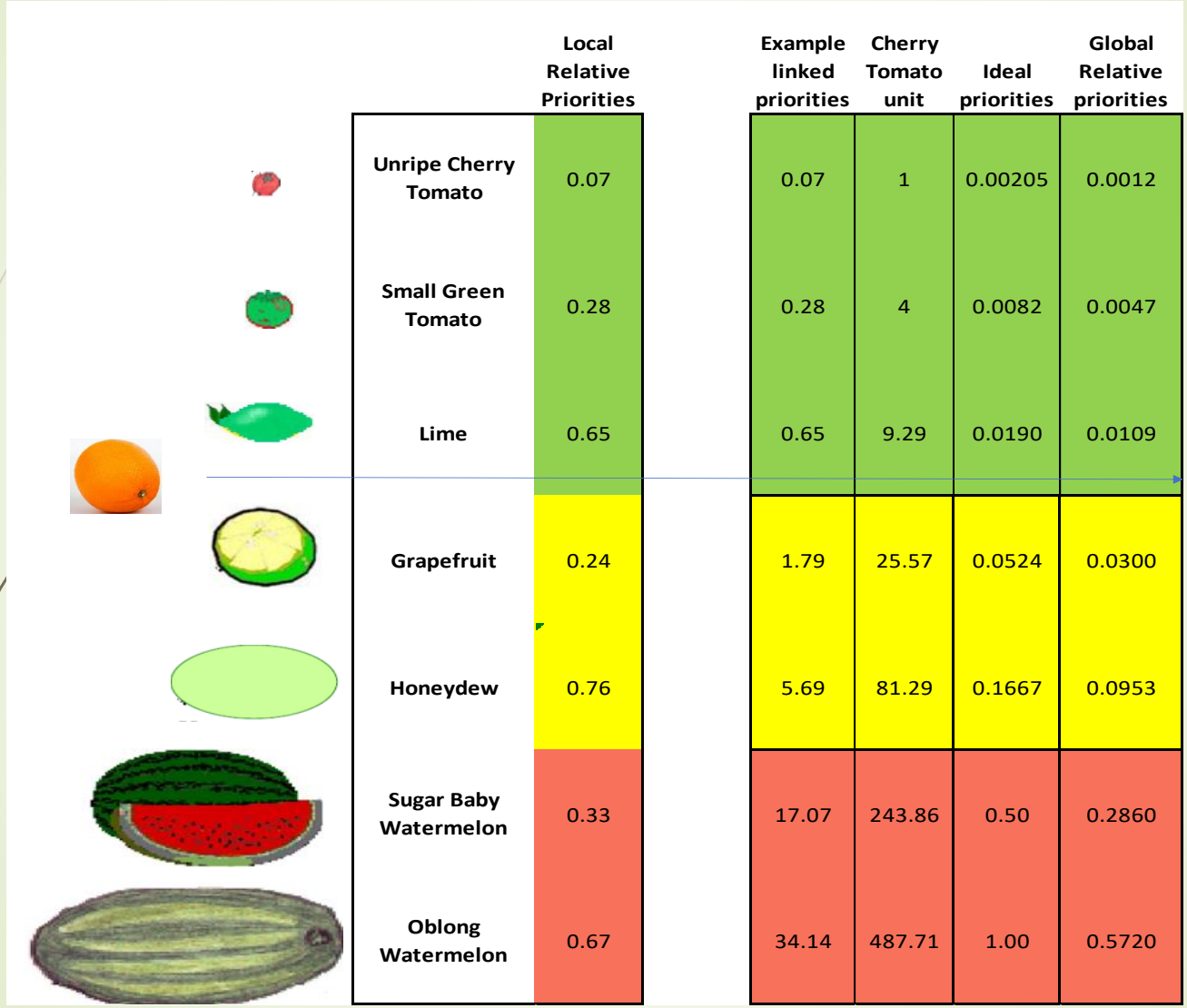
Scales from measurement

- In 2004, Tom Saaty wrote a profound paper called “Scales from Measurement Not Measurement from Scales”. This was only published as a conference paper.
- Claudio Garuti (2017) is one person who knows the significance of this paper. He published a paper about it, giving his reflections.
- Tom was referring to AHP scales being derived from the ratio measurements of paired comparisons, not from measurements from some predetermined scale.
- The ability to build evidentiary scales where no prior scales existed is part of the intuitive magic of AHP.
- But what if you could reverse the process and use the evidentiary scales as fixed standards for measurement.

Freezing AHP scales

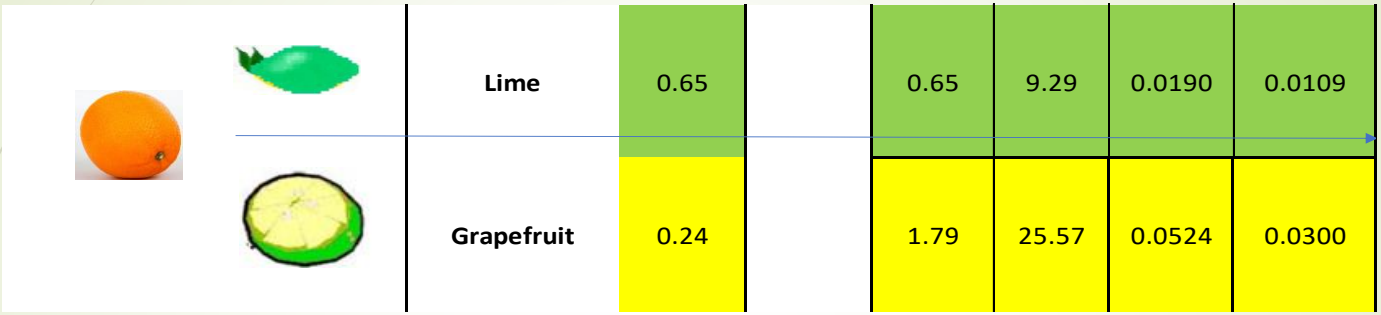
- Benchmark measurement is based upon the idea that once you have built and AHP scale, you can freeze it and allow it to become a measurement scale for other objects.
- In other words, scales with fixed units enable measurement from scales.
- Let us look at our fruit scale to see how we can add additional fruits.

Where would an orange fit?



	Local Relative Priorities	Example linked priorities	Cherry Tomato unit	Ideal priorities	Global Relative priorities
Unripe Cherry Tomato	0.07	0.07	1	0.00205	0.0012
Small Green Tomato	0.28	0.28	4	0.0082	0.0047
Lime	0.65	0.65	9.29	0.0190	0.0109
Grapefruit	0.24	1.79	25.57	0.0524	0.0300
Honeydew	0.76	5.69	81.29	0.1667	0.0953
Sugar Baby Watermelon	0.33	17.07	243.86	0.50	0.2860
Oblong Watermelon	0.67	34.14	487.71	1.00	0.5720

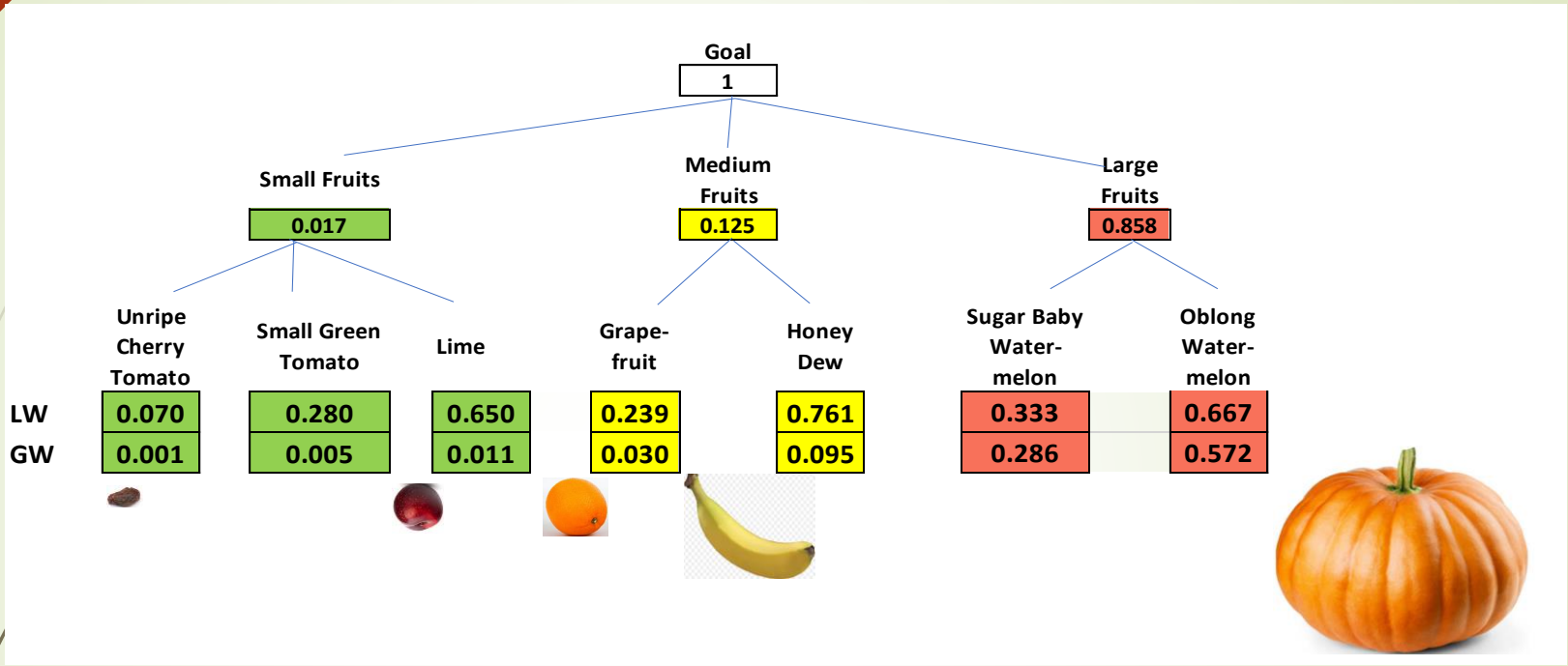
How can we determine where orange fits between lime and grapefruit?



Lime	0.65	0.65	9.29	0.0190	0.0109
Grapefruit	0.24	1.79	25.57	0.0524	0.0300

- Interpolation: On the interval between lime and grapefruit, estimate the percentage orange is greater than lime.
- Paired comparison:
  - Grapefruit is bigger than orange. How many times?
  - Orange is bigger than lime. How many times?

Bottom up Fruit Benchmark



- Interpolation or paired comparisons can be used to position many different types of fruits on the benchmark scales. With the scales frozen to a specific unit, there is no need to do excessive comparisons.

- In this manner, scales from PRM measurements can be frozen as benchmarks and then used for measuring many additional items.

Theme 8: Benefits and Costs

- What if I gave you the following Benefit and Cost priorities for three different projects.

	Aggregate Benefit priorties	Aggregate Cost priorties	Benefit/Cost Ratio
Project A	0.42	0.54	0.78
Project B	0.37	0.28	1.32
Project C	0.21	0.18	1.17
	1	1	

Which project would you select?

Benefits and Costs

- What if I told you that the benefit and Cost priorities came from the following aggregate Benefits and Costs measured in millions of dollars.

	Aggregate Benefit priorties	Aggregate Cost priorties	Benefit/Cost Ratio	Aggregate Benefits in million $	Aggregate Costs in million $	Benefit$/Cost$ Ratio
Project A	0.42	0.54	0.78	4.2	8.1	0.52
Project B	0.37	0.28	1.32	3.7	4.2	0.88
Project C	0.21	0.18	1.17	2.1	2.7	0.78
	1	1				

- Which project would you select?
- A one here is not equal to a one there.
- Diederik Wijnmalen (2007) has studied this issue.

Theme 9: Similarity of Hierarchies and Networks

Comparison of AHP and ANP

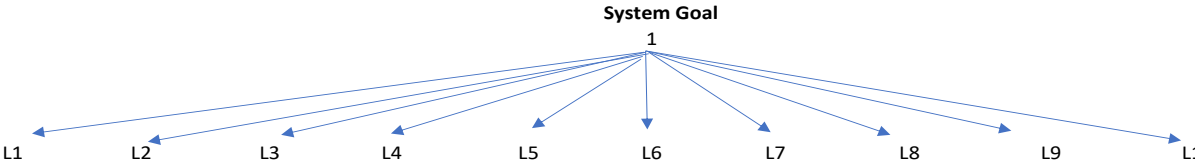
Features	AHP	ANP
Structure		
a. Connections	hierarchy of elements and clusters connected by lines	network of elements and clusters connected by arrows that indicate direction of influence.
b. Effects	line shows relationship, not direction of influence.	source element (or cluster) of the arrow influences the destination element
Local weights from PRM		
a. Placement of priorities	placed in clusters on each level of the hierarchy	placed in clusters in a column of the unweighted supermatrix
b. Normalization to unity	elements in each cluster sums to unity	elements in each cluster sums to unity
c. Sums across clusters	sum of a level equals the number of clusters	sum of a column equals the number of clusters
Global weights		
a. Weighting of priorities	local weights times the parent global weight	local weights times the cluster importance in the column.
b. Resulting Relationship	relative importance of the element in the hierarchy	relative influence of the of the element in the column
c. Sums across clusters	sum of global weights at each level equals unity	sum of a column equals unity
Overall solution		
a. Aggregation procedure	sum of global priorities for each element in the hierarchy	stable result from matrix multiplication of the weighted supermatrix
b. Unit of final solution	total of global weights equals unity.	total of stable weights for all network elements equals unity
c. Source of solution	single or multiple hierarchies	supermatrix or a super hierarchy (weighted multiple hierarchies)

Supermatrices can be viewed as Column Hierarchies

This is how a Supermatrix can be viewed as a series of column hierarchies within a larger hierarchy that forms the network system.

- Below is the unweighted Supermatrix depicted within the larger hierarchy. Links L1 to L10 represent weights for the columns that will link the columns together.

Unweighted Supermatrix as Column Hierarchies



	Big/ Small Package	Variety Package	Melon Package	Unripe Cherry Tomato	Small Green Tomato	Lime	Grape-fruit	Honey Dew	Sugar Baby Water-melon	Oblong Water-melon
Big/Small Package	0	0	0	1	0	0.500	0.500	0.143	0	0.500
Variety Package	0	0	0	0	1	0.500	0.500	0.571	0.500	0
Melon Package	0	0	0	0	0	0	0	0.286	0.500	0.500
Unripe Cherry Tomato	0.301	0	0	0	0	0	0	0	0	0
Small Green Tomato	0	0.301	0	0	0	0	0	0	0	0
Lime	0.699	0.699	0	0	0	0	0	0	0	0
Grapefruit	0.386	0.136	0	0	0	0	0	0	0	0
Honey Dew	0.614	0.864	1	0	0	0	0	0	0	0
Sugar Baby Watermelon	0	1	0.333	0	0	0	0	0	0	0
Oblong Watermelon	1	0	0.667	0	0	0	0	0	0	0
Sum	3	3	2	1	1	1	1	1	1	1

Some Conclusions

- "AHP/ANP can be simplified into 1 sentence: "A method that links/connects/aggregates objects together using priorities from relative comparisons." (Orrin Cooper)
- Ratio scales derived from PRM measurements is like magic. The scales "make evidence" where none existed beforehand (Yoichi Iida)
- Any change in priorities via multiplication by some positive constant transforms the measurements to a new scale that has a different unit of measure.
- To get a better understanding of what is happening, follow the unit.
- To get the full power of AHP/ANP use it in its ratio form that gives the intensity of one object over another.

Call to Action

- The unit is there. It will not go away. We need to recognize and address it.
- “A one here is not equal to a one there” (Sajjad Zahir).
- The dilemma is that the unit is obscure. Herein we have shown (1) where mistakes can be made with an obscure unit and (2) where beneficial procedures can occur if the unit is recognized.
- The AHP/ANP community is called to action.
- To continue to ignore the issue of units is dysfunctional
- My hope for this session is that I have created functional inconsistency in your mind about the advantages and dilemmas of the unit.
- Functional inconsistency is what has driven my academic career.
- Functional inconsistency can catapult AHP/ANP to new frontiers.

Bright Future!

Thank you!

Questions and Discussion

Related Work

1. **Do we need to know natural zero to do comparisons? (Wedley, 2007; Wedley & Choo, 2008)**
2. **Do we need to do all comparisons? (Wedley, 1993; Wedley, 2009)**
3. **Do AHP priorities have a unit? (Wedley & Choo, 2011; Choo, Schoner & Wedley, 1999; Schoner & Wedley, 1989; Wedley, Schoner & Choo; 1993)**
4. **How do you get the full benefit of ratio intensities? (Choo & Wedley 2010; Choo, Schoner, & Wedley, 1999; Wijnmalen & Wedley, 2008)**
5. **Are criteria weights dependent upon alternatives? (Schoner & Wedley, 1989; Schoner, Wedley & Choo, 1993, Wedley, Schoner & Choo, 1993)**
6. **Can criteria weights be independent of alternatives? (Wedley, Schoner & Choo, 1996, Wedley & Choo, 2009)**
7. **Can derived scales be used as scales for measurement? (Wedley, Schoner & Choo, 1996)**
8. **Do benefits and cost priorities have to be commensurate? (Wedley, Choo & Schoner, 2001)**
9. **Can a unit interpretation be applied to the supermatrix? (Cooper & Wedley, 2020)**