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## **Investigation for Carrying Incomplete Pairwise Comparisons in AHP**

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**Summary:** *This paper report results of numerical experimental comparisons with random number for carrying out incomplete pairwise comparisons. In this report, Harker method (HM) and Logarithmic least square method (LLSM) are compared. Several methods of selecting comparing pairs are used for numerical experiments*

### **1. Introduction**

The number of pairwise comparisons necessary in AHP is square order of the number of the alternatives. So the great number of pairwise comparisons is necessary when there are many numbers of alternatives. This paper report results of numerical experimental comparisons with random number for carrying out incomplete pairwise comparisons.

### **2. Algorithms**

Several algorithms for calculation from incomplete comparisons have been proposed. Among them, Harker method (HM) and Logarithmic least square method (LLSM) are compared .

### **3. Selecting Methods**

Followings methods of selecting comparing pairs are used for numerical experiments.:

- Front and back m alternatives,
- Hierarchical selected by order of numbers
- Hierarchical selected by order of value.

### **3. Numerical experiments**

From numerical experiments with random numbers, following results are gained.

- LLSM is slightly superior to HM.
- In the examined methods of selecting comparing pairs, hierarchical selected by order of value is the best.
- Difference which algorithm is used for calculation is smaller than difference which method of selecting comparing pairs is used.

Table 1. Correlation coefficient between incomplete pairwise comparison and complete pairwise comparison (S=1.3)

Selecting method	algorithm	15 alternatives	20 alternatives	25 alternatives	30 alternatives
correlation coefficients					
Front and back 3 alternatives	HM	0.9706 (0.9693 0.9720)	0.9553 (0.9534 0.9571)	0.9374 (0.9346 0.9402)	0.9174 (0.9134 0.9214)
	LLSM	0.9732 (0.9719 0.9744)	0.9646 (0.9633 0.9659)	0.9555 (0.9538 0.9572)	0.9480 (0.9463 0.9498)
Hierarchical selected by order of numbers	HM	0.9540 (0.9518 0.9563)	0.9562 (0.9545 0.9579)	0.9584 (0.9571 0.9598)	0.9587 (0.9575 0.9599)
	LLSM	0.9618 (0.9600 0.9635)	0.9627 (0.9613 0.9640)	0.9633 (0.9620 0.9645)	0.9636 (0.9625 0.9646)
Hierarchical selected by order of value	HM	0.9679 (0.9661 0.9697)	0.9713 (0.9704 0.9723)	0.9726 (0.9717 0.9735)	0.9734 (0.9727 0.9741)
	LLSM	0.9732 (0.9721 0.9744)	0.9748 (0.9740 0.9757)	0.9758 (0.9751 0.9766)	0.9768 (0.9762 0.9774)
Hierarchical evaluation		0.9378 (0.9347 0.9410)	0.9373 (0.9349 0.9396)	0.9402 (0.9380 0.9423)	0.9411 (0.9392 0.9430)
Spearman's rank correlation coefficients					
Front and back 3 alternatives	HM	0.9568 (0.9548 0.9589)	0.9468 (0.9445 0.9492)	0.9355 (0.9328 0.9381)	0.9246 (0.9212 0.9280)
	LLSM	0.9591 (0.9571 0.9610)	0.9549 (0.9529 0.9568)	0.9489 (0.9468 0.9509)	0.9458 (0.9440 0.9477)
Hierarchical selected by order of numbers	HM	0.9411 (0.9382 0.9440)	0.9492 (0.9471 0.9512)	0.9523 (0.9505 0.9540)	0.9555 (0.9540 0.9569)
	LLSM	0.9486 (0.9461 0.9511)	0.9538 (0.9519 0.9557)	0.9563 (0.9546 0.9579)	0.9598 (0.9585 0.9611)
Hierarchical selected by order of value	HM	0.9526 (0.9502 0.9550)	0.9611 (0.9595 0.9626)	0.9640 (0.9627 0.9652)	0.9677 (0.9667 0.9687)
	LLSM	0.9582 (0.9562 0.9603)	0.9643 (0.9629 0.9657)	0.9666 (0.9655 0.9678)	0.9702 (0.9693 0.9712)
Hierarchical evaluation		0.9193 (0.9151 0.9234)	0.9225 (0.9193 0.9257)	0.9277 (0.9250 0.9303)	0.9330 (0.9307 0.9353)

Upper: mean,

Lower: 95% confidence Interval

Table 2. Correlation coefficient between incomplete pairwise comparison and complete pairwise comparison (S=1.5)

Selecting method	algorithm	15 alternatives	20 alternatives	25 alternatives	30 alternatives
correlation coefficients					
Front and back 3 alternatives	HM	0.9490 (0.9467 0.9513)	0.9261 (0.9233 0.9289)	0.8981 (0.8943 0.9019)	0.8721 (0.8676 0.8766)
	LLSM	0.9546 (0.9527 0.9565)	0.9408 (0.9388 0.9428)	0.9259 (0.9234 0.9285)	0.9137 (0.9110 0.9164)
Hierarchical selected by order of numbers	HM	0.9246 (0.9215 0.9278)	0.9261 (0.9235 0.9288)	0.9271 (0.9248 0.9294)	0.9261 (0.9239 0.9283)
	LLSM	0.9358 (0.9332 0.9384)	0.9373 (0.9352 0.9393)	0.9368 (0.9348 0.9388)	0.9362 (0.9344 0.9380)
Hierarchical selected by order of value	HM	0.9422 (0.9396 0.9448)	0.9458 (0.9438 0.9477)	0.9468 (0.9452 0.9484)	0.9486 (0.9471 0.9501)
	LLSM	0.9500 (0.9478 0.9523)	0.9529 (0.9513 0.9546)	0.9533 (0.9518 0.9547)	0.9551 (0.9538 0.9564)
Hierarchical evaluation		(0.8826 0.8932)	0.8891 (0.8849 0.8933)	0.8902 (0.8865 0.8939)	0.8941 (0.8909 0.8973)
Spearman's rank correlation coefficients					
Front and back 3 alternatives	HM	0.9369 (0.9339 0.9398)	0.9227 (0.9195 0.9259)	0.9036 (0.8999 0.9073)	0.8897 (0.8858 0.8937)
	LLSM	0.9407 (0.9215 0.9272)	0.9333 (0.9307 0.9360)	0.9243 (0.9468 0.9509)	0.9189 (0.9161 0.9216)
Hierarchical selected by order of numbers	HM	0.9155 (0.9116 0.9193)	0.9231 (0.9200 0.9261)	0.9259 (0.9233 0.9284)	0.9290 (0.9268 0.9312)
	LLSM	0.9240 (0.9207 0.9274)	0.9314 (0.9287 0.9341)	0.9336 (0.9312 0.9360)	0.9376 (0.9356 0.9395)
Hierarchical selected by order of value	HM	0.9288 (0.9253 0.9324)	0.9375 (0.9350 0.9401)	0.9421 (0.9401 0.9441)	0.9471 (0.9453 0.9488)
	LLSM	0.9348 (0.9316 0.9380)	0.9427 (0.9404 0.9450)	0.9464 (0.9446 0.9483)	0.9512 (0.9496 0.9528)
Hierarchical evaluation		(0.8667 0.8790)	0.8803 (0.8757 0.8850)	0.8835 (0.8792 0.8878)	0.8913 (0.8877 0.8949)

Upper: mean,

Lower: 95% confidence Interval

## References

Saaty, T. L. (1980) *The Analytic Hierarchy Process*, McGraw-Hill.

Harker: P. T. (1987), "Incomplete pairwise comparisons in the Analytic Hierarchy Process," *Mathematical Modeling*, 9, 837–848.