BRIEF PARALLELISM AMONG THE ANALYTIC NETWORK PROCESS (ANP) & FRACTAL GEOMETRY

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Keywords: ANP, Fractals, system representation, Parallelisms, History

Summary: *The aim of this work is to show the parallelisms and analogies that exist in modeling and measuring of dependence and feedback processes, in physical and in decision making processes, this is, to compare among the scales of measurement of the physical world (geometry) and the scales of measurement of the human being internal decision process, in other words, the brain's internal generation of relative measure scale.*

1. Introduction

About the :"brain internal generation of relative measure scales", we can quote the following; *"Trading off is the human form of doing things. Nature even makes tradeoffs in the human body by setting up hormones that operate on compensation principles. The brain is empowered to supervise equilibrium by ordering or blocking the secretion of one substance or another to orchestrate the workings of the entire body. I believe that the brain itself must operate on a relative principle, as it has no stored scales of measurement to determine the absolute amount of each substance needed to maintain balance".* (Saaty, 1996).

As a representative example of the above-mentioned, we can add that the values described as "normal" in the medical literature regarding the lipoproteins, represent a range that defines the population's statistical average, therefore, individuals exist with levels in the lower bound of the accepted range, and others in the upper bound, with differences up to 50% in their values and being physiologically healthy. Any intent of modifying these values in a pharmacological way or by means of diets, may result in a decompensation that shows up by driving hormonal answers to pathological symptomatology (hyper or hypoglycemia, variations in the triglicerids, etc..).

Indeed, "*the creative mind, (it) is driven by the imagination and our ability and reason ability. Imagination is fragmentary and needs purpose and cohesion unity. In science, imagination always precedes unity. Synthesizing its creations takes time. A long-standing and troubling observation to me has been the fragmentary and evanescent nature of our knowledge. This is due both to the diversity of our experience and to an absence of goal-oriented thought structures-hierarchies-needed to knowledge from one set of goals to a higher set and ultimately for our survival".* (Saaty, 1996) .

As an intermediate objective of this work, we expose the basic concepts of measurement interpretation and quantification, and their development in the history (Euclidean, Cartesian and Fractal geometry). Consequently, the basic concepts of metric, and their interpretation are exposed, in the physical world (distance concept), and in the decision making world, as well as the existent analogies among the metric and the Analytic Hierarchy Process (AHP) axioms.

On the other hand, this work also considers some of the mathematical parallelisms that exist in the representation of the reality of both systems (ANP & Fractal geometry) and in its modeling processes, as: limits, convergence, stability, attractor, and still others like errors, randomness, consistencies and their physical interpretations, which are showed in a graphical way with simple explanations and interpretations.

2.- Basic Concepts of Geometry in History

i. - Basic Concepts Of Interpretation And Quantification Of The Measure, And Their Development In The Human History.

The first measurement principles, with an appropriate logical structure, appeared in the human history in the old Greece, mainly with the great Euclid and his principles of representative geometry, where the objects were idealized (like in Plato's vision), and represented geometrically as stand-alone abstract figures with no connections between them and the reality, as illustrated in Figure 1.

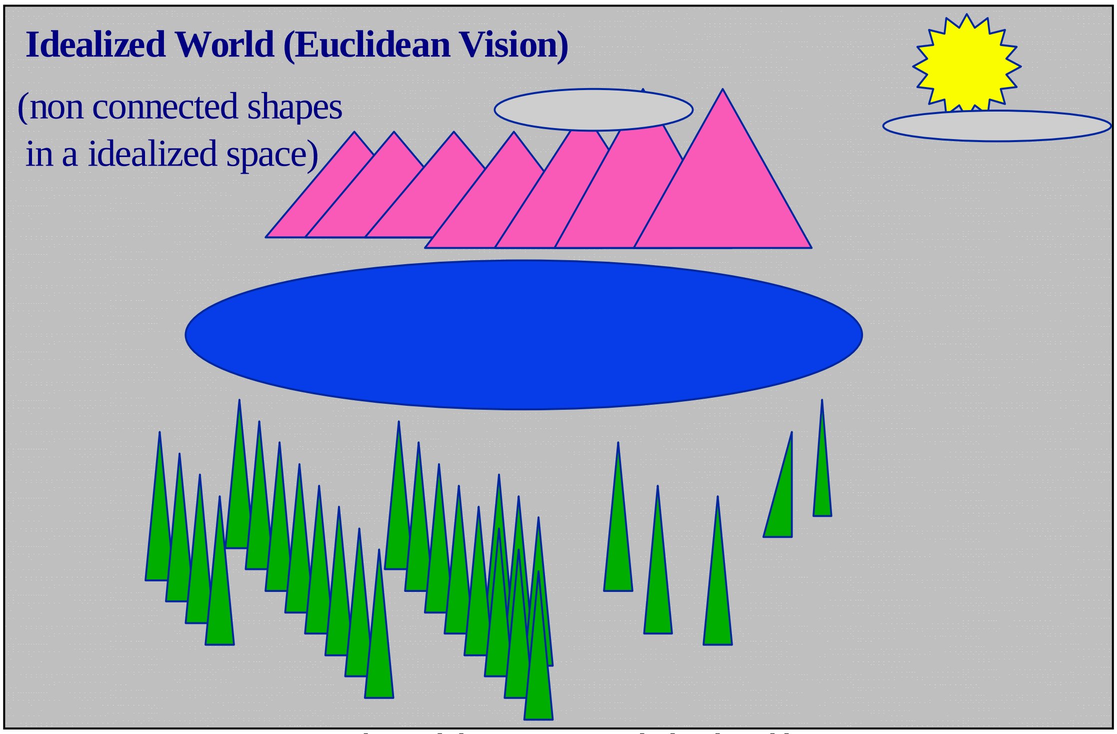


Figure 1: The Euclidean Vision, an idealized world vision

2000 years later, a new vision of the world was generated which affected the way of understanding and measuring the world. This time, the famous French philosopher and mathematician René Descartes is the one commissioned to give a representation of this new vision, by means of its well-known Cartesian coordinated linear system with an absolute point of reference (0,0). Here, the idea is to connect all the elements to each other by means of a mesh of invisible points which share a common origin point (0,0). Doing so, it is feasible to generate relative measurements among the elements, and even more important, to generate an absolute measurement system able to assign a length (absolute measurement) in relation to a single origin point.
This vision, used so far as the standard or normal form of reality representation, can be graphically showed through the following figure, (Figure 2: Cartesian vision):

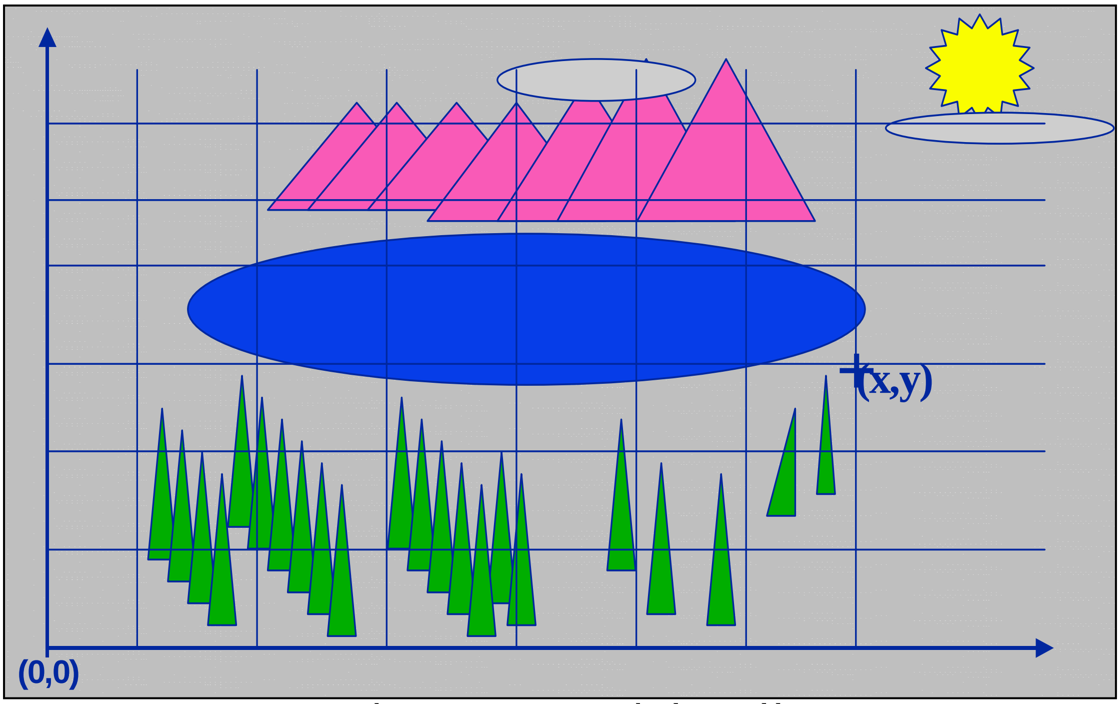


Figure 2: The Cartesian vision, an absolute world vision.

At this point of the story, we would like to make a parenthesis just to highlight the relationship between the old Greek's culture and Euclid and Plato's world vision, (an abstract and ideal world), and by the other side, the relationship between the Descartes world vision and the absolutist culture (religious dominance) of those times.

The Cartesian vision of the world, lasts until nowadays, or rather until the computers era, (where old measurement problems like the punctual aberrations of the continuous mathematical models were believed to be just a mathematical impasse that had to be solved sooner or later (Hilbert, Whitehead, Russell). The computer era reestablished this mathematical problems and at the same time, helped to conceive and reproduce a new sight of the world, completely interconnected and interdependent, where the elements don't exist by themselves and no absolute and/or close systems can capture the richness of the world, hence where a new reality vision much more complex and rich takes place, one of holistic and relativistic kind (Einstein, Godel, Mandelbrot). At the same time, terms like fractional dimensions or completely relative measurement (any point can be the origin, but not one is really the origin) begin to appear, up to more "bizarre" things like: a straight line is not necessarily the shortest path between two points, or that the measurement of a distance is not an absolute value but, it depends on the scale used or under what conditions it is made.

Regarding the measurements in relativity, we can mention the following example, frequently used in literature: two observers in relative movement regard each other. Suppose that one of them holds two poles, with the same electrical charge one in each hand. The one that hold the charges, simply measures the existing repulsion force among them, while the observer in relative movement, measures the repulsion force, plus an attraction force induced by the generated magnetic field due to the relative movement of the charges. If the first observer sets the charges free, both of them will measure different movement speeds, and moreover, different elapsed time for the charges to reach a certain place.
The question that arises is: which of the two observers is seeing the "true" reality? These and other many physical phenomena totally escape to the traditional way of seeing, representing and measuring things. Why is it so different with the "natural form" in which we were educated since childhood and to which we are so used to?
This new way of representing the reality is completely different, it offers a much richer vision, but at the same time, a much more complex and more interdependent one. It is no longer possible to study a portion of a system independently of the rest, as the need arises to analyze their dependences and feedbacks, as well as to take into account the border conditions (constrains) and the relative measurement rules that take place among the observer and the observed (Heisenberg, Penrose). To this respect, an interesting link with the classical definition of reality (constructivistic reality) can be made:

"*In the western cultural tradition in which modern science and the technology have arisen, we speak, in the daily life, of reality and real things, as a domain of entities that exist independently of what we, as observers, do. Even more, we act and speak, both colloquial and technically, as if the knowledge meant to be able to make reference to such things as independent entities. The flowing of the normal daily experiences in which we find objects as if they were there independently of what we do, seems to confirm this. I want to change this way of thinking, looking for deeper reflections in the explanation if the biology of observing, about the consequences of accepting the operational separation of the experience and the explanation of the experience.*" (Maturana, 1997). "The Objectivity, An Argument to Force":

An example of this kind of vision and in the spirit of the precedent examples is shown in Figure 3, (Figure 3: Fractal vision).

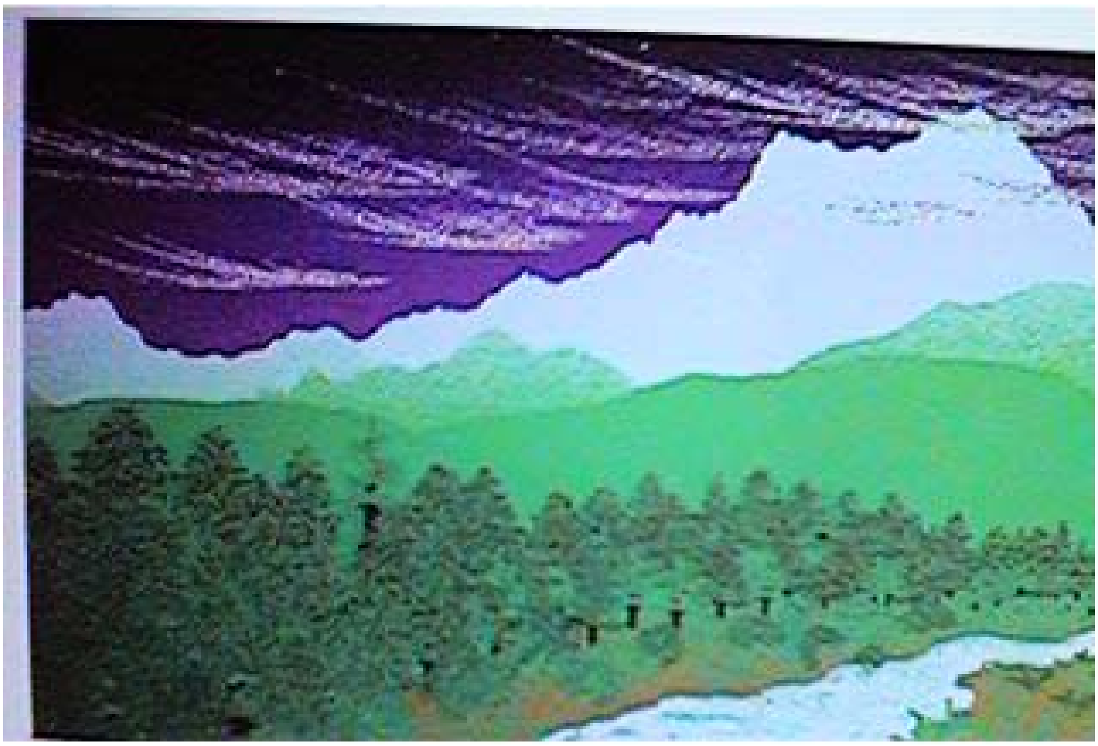


Figure 3: The Fractal Vision, a highly interconnected (holistic) and relativistic world vision

ii. - Metric, Topology and Decision, the Axiomatic Basic Concepts

To establish the parallelism between the ANP and the Fractal geometry, it is necessary to give a brief summary of the measure theory, the AHP/ANP axioms and their relationships, since both systems are based on the measurement concepts, and although their applications areas differ, their implications are common.

Metric Axioms :
A metric space is defined as a set X and a Real function d (called distance) over XxX → R, such that:
1) d(x,y) >= 0, d(x,y) = 0 if and only if x = y (non negativity)
2) d(x,y) = d(y,x) (symmetry),
3) d(x,y) <= d(x,z) + d(z,y) (triangular inequality)
Where all x, y and z belong to X.

We also have:
* If E and F are two non empty groups, subsets of the metric space X, then the distance between them is defined as: d(E,F) = Min [d(x,y), with x belonging to E, and y belonging to F] (minimum distance).

By the other side, we have the 4 basic AHP Axioms:

Axiom 1: Reciprocity
For all systems based in pairwise comparisons, this axiom points out that the intensity of preference of Ai over Aj is inverse to that of Aj over Ai. That is:
Given 2 alternatives (Ai, Aj) of AxA, the preference intensity P on a property c_k, is inverse to the preference intensity of (Aj, Ai) over the same property c_k => Pc(Ai,Aj) = 1/Pc(Aj,Ai). For all Ai, Aj belonging to A.

Axiom 2: Homogeneity
This axiom deals with the human precision in the measure, remarking the fact that the compared elements should have the same order of magnitude.
=> Ai <10*Aj, For all Ai, Aj belonging to A. and regarding a property c_k.

Axiom 3: Dependence
This axiom determines the kind of internal or external dependence among the elements, according to the fact that the variables may belong to the same cluster (internal dependence) or to different clusters (external dependence). In the particular case of hierarchies (AHP), this analysis can be used to try to control the dependence among the alternatives and, whenever possible, of the alternatives with the criteria and the criteria with themselves.

Axiom 4: Expectations
This axiom states that all the expectations should be represented in the system in terms of properties and alternatives. The determined priorities are expected to be compatible with these expectations.
=> The aggregation and/or elimination of alternatives (near copies) should be accompanied with the elimination and/or aggregation of new measurement properties in the system, if we wish to keep the original stability of the system. In other words Aj ε {A} is a real alternative if and only if we have no other alternative Ai ε {A} completely equivalent to Aj. This means: Pc_k (Ai, Aj) = 1 for all k. (C_k, is the set of the criteria).

If a parallelism is carried out among these two groups of axioms, there are several coincidences, due to the fact that both share the objective of building a measurement or a metric space, although this is done in very different environments. Lets go a bit deeper:

The first axiom of AHP presents a similar requirement as the first metric axiom, which establishes the non negativity of the measure inside a defined space. In the same way that it doesn't make sense to speak of negative distance solutions, neither speaking of negative preference solutions makes sense. We wish to point out that this notion is not in opposition with the idea of negative priorities, since this last idea arises from the subtracting concept, dealing with elements in two different spaces, for example, benefits and costs. (Korhonen y Topdagi (2002), "…about the separation of the utility scales").

The second axiom of the AHP deals with the measurement precision, and despite the fact of not having a direct relationship with any axiom of the measurement theory, it is related with the need to keep what is called the consistency of the measure. All measures require to maintain consistency inside the measurement space is used, and this is achieved by means of the symmetry and triangular inequality axioms. Similarly the axiom of the homogeneity offers a tool to keep under control the consistency inside the measurement space that the decision maker is generating.

A third equivalence in the construction of the measurement scales can be seen in the axiom 4 of the AHP, the expectations. This axiom is related with the construction of the measurement spaces instead of the measure itself, it means that, to obtain a useful measure it is first necessary to build a complete space. (The metric spaces, are defined as complete spaces).
In simple words, if the expectations are not complete it can mean either: some properties (dimensions) have not been included, or an existing property or alternative is a copy of another (the space has some dimensions that are linearly dependent of others, i.e. the Kernel space is smaller than the one needed to generate a well defined metric).

3.- Parallelisms Among the Models and its Algebraic and Geometric Representations

After this necessary, although a little arid, parenthesis regarding the axiomatic parallelisms in the measurement definition, it is possible to explain the reasons, by means of which two completely different measurement models (Fractal and AHP/ANP), that try to solve different problems, are capable of having these similarities, not only of structural type but also regarding their consequences.

To illustrate this situation, a parallelism of structural type is shown, where both models use the same basic principle of composing information.
In the case of the hierarchical models (AHP), the composition principle is not generated (in opposition to what some people think), in a simple additive way, like in: Σ Ai x wi, but it rather corresponds to a multilinear form like in: $x_{11} * x_{12} * \ldots * x_{1p} + x_{21} * x_{22} * \ldots * x_{2p} + x_{i1} \ldots * x_{ip}$
or $\Sigma_i(\Pi_p(x^i{}_p))$ being i, the number of terminals (covers) criteria, and p the number of levels of the hierarchy minus one.

This represents the simplest form of a non linear function, and its density in the complete spaces (with defined metric), assures the capacity to represent the problem in the used required scale (depth or zooming function).

Fractals on the other hand, also use the same idea of composing the information: they do so, by composing elementary figures in a progressive way as to generate complex figures.
For example, a coastline representation, can be carried out by means of the composition or aggregation of triangles that are built to trisect the sides of the resulting figure, if this operation is repeated many times (and the internal lines are eliminated), one might, depending on the boundary conditions, obtain a representation of the coastline at any desired scale.

As a graphic example of the above-mentioned, the construction of a coastline by means of infinite superimposed triangular elementary blocks is shown. (Figure 4: Koch Coastline Function)

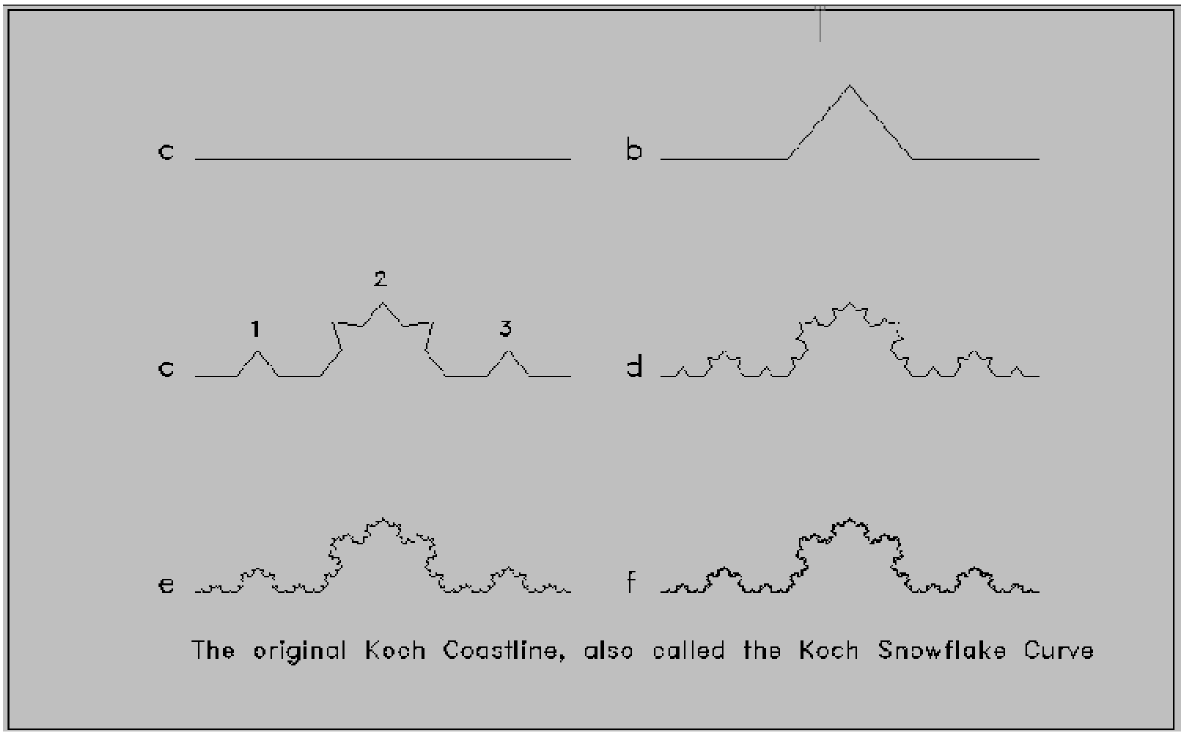


Figure 4: Koch Coastline Function

The beauty of these composition systems, is that they allow the user, to go as close as needed to represent the problem. This is like having a structure that allows zooming in to represent, in the required scale, either the space and its phenomena (this situation is found in both fractal geometry and

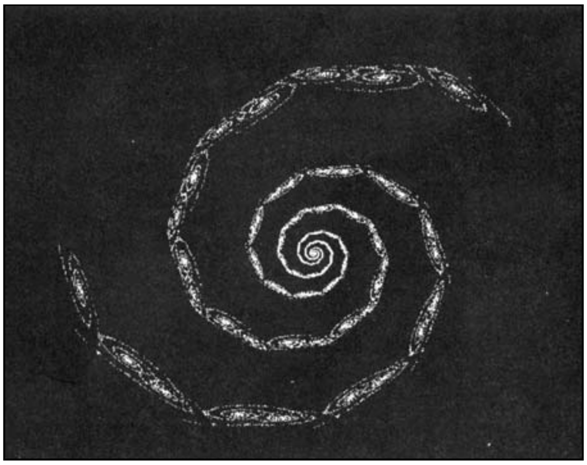
AHP/ANP decision modeling). In both of them the complexity is represented by the structured composition of its basic elements (elementary geometry figures, criteria or cluster's nodes).

Other interesting parallelism between these representation structures (systems), deals with the necessity of uncertainty or randomness handling. In both cases, in order to achieve a better reality approach, it is necessary to allow for inconsistency or randomness.
In the AHP/ANP case, this is achieved by allowing a certain degree of inconsistency in the matrix comparison, which gives the flexibility for input and accommodation of new information without corrupting the original system. This is an absolutely necessary condition in any open system, since in many cases it enables to improve the quality of the original AHP/ANP thinking process model.
In the Fractal case, adding a degree of randomness in the basic system, allows a better approach to the modeling situation, as it can be visualize in the following figures: (Figures 5a and 5b):

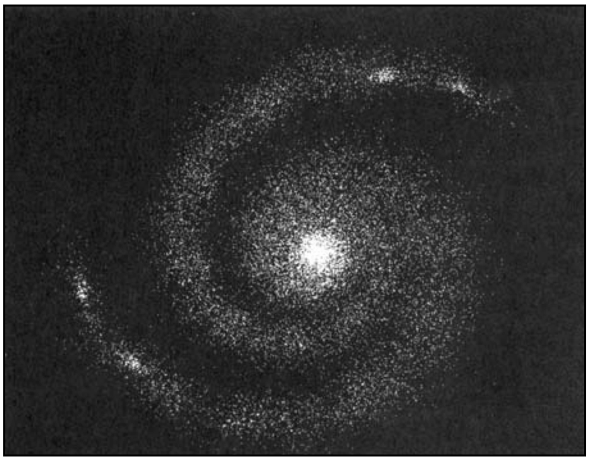
ERROR HANDLING ⇔ HANDLING OF NEW INFORMATION IN OPEN SYSTEMS

AHP/ANP => Inconsistency **(needed to represent the real thinking process)**
FRACTAL => Randomness **(needed to recreate the real world)**

Figure 5a: Handling of Information in Open Systems



Galaxy without Fractal randomness



Galaxy with Fractal randomness

Figures 5b: Inconsistency and Randomness

Other analogies that can be found in the literature regarding the behavior and representation of these models, are:

Resonance Effect:
It is possible for a variable or criterion that initially presents a low weight or importance, to be connected with another variable of high weight, and due to the interaction of this two elements it can happen that, by a resonance effect, the initially low importance variable increases its importance dramatically, (this should make us meditate on the simplifications that sometimes are carried out in the AHP models by dropping some *"low weight variables"*, without considering their eventual interactions with other variables of the model).

On the other side, regarding the fractal geometry and its algebraic representation (chaos theory), this effect is quite common. For example, the meteorological well-known butterfly effect (Lorenz,1961) which refers to the possibility that the flapping of butterfly wings in the Japan sea (extremely low

weight variable), produces a typhoon in the Hawaii coasts, due to a continuous interaction of resonance and reverberation of this variable with others of more weight that possibly amplify its importance to unsuspected levels. That’s why specifying the state of the weather any day in the future is so complicated. As this temporary horizon increases its distance, so does the threshold, and the interactions of tiny variables increase in an exponential form. It is necessary to point out that this effect doesn't attempt against the global stability of the system, (holistic vision) because although, it can be predicted (with a high success probability) that it will be cold next winter and it will rain, nothing can be said regarding one specific day of that winter.
Plenty of examples like these appear in the literature, in fact we can find applications of AHP/ANP in economy, environment, social and even in sport forecast. For long term conditions, prediction applications exist in the same areas, mainly with dynamical systems (chaotic models) and their graphical representation, the fractals . (As an example, let see the chaotic models for economic index behavior for long periods).

Consequences

It is interesting to detail the parallelisms and the positive and negative consequences of these prediction models. As an illustrative example, a chart of equivalencies is shown:

ANP	FRACTALS
More input data required	All steps 1,2,3 ...n must be executed. No simplifications or extrapolation shortcuts allowed.
More Final Results (than those originally looked for)	Holistic vision, (although only part of it was required)
Steady vectors (eigenvectors)	Steady Points (attractors)
More than one eigenvector in the Supermatrix (oscillant solution)	More than one attractor in the Equation (oscillant solution)

Figure 6: Chart of equivalencies

This chart of equivalencies refers to the computational use of these models and its consequences. For instance, in the ANP the comparison of all the related variables is required to determine their final behavior (dependence). In the same way, in the fractal world extrapolations cannot be made, point 3 cannot be drawn without passing through the previous points 1 and 2.

On the other side, as more data than the required by the system was entered more results (correlations) appear, than those originally looked for. In fact many of these results (lateral or not expected) do not have a simple interpretation. In the same way, the Fractal image must necessarily be completely built even when only part of it is required.

In the ANP, steady solution vectors arise (eigenvectors), as well as in the case of Fractals, where steady points of attraction arise (attractors). But in both systems, steady dipolic points may arise, i.e., more than one solution (eigenvector or attractor) may appear, and the solution may oscillate between these two points (scenarios).

As a last comment regarding the behavior of these interdependent and chaotic systems, it can be stated that in both models (or measurement constructions) the results are: *Highly Dependent On The Initial Conditions And On System’s Degree Of Feedback.* Despite the fact that this conclusion seems to be “*obvious*”, it is a very important one, since this is a strongly expected result that has to be accomplished in both cases.

Finally, a global parallelism regarding the time construction of the physical measurements (geometry) and the construction of the decision measurements is shown in the next figure (Figure 7):

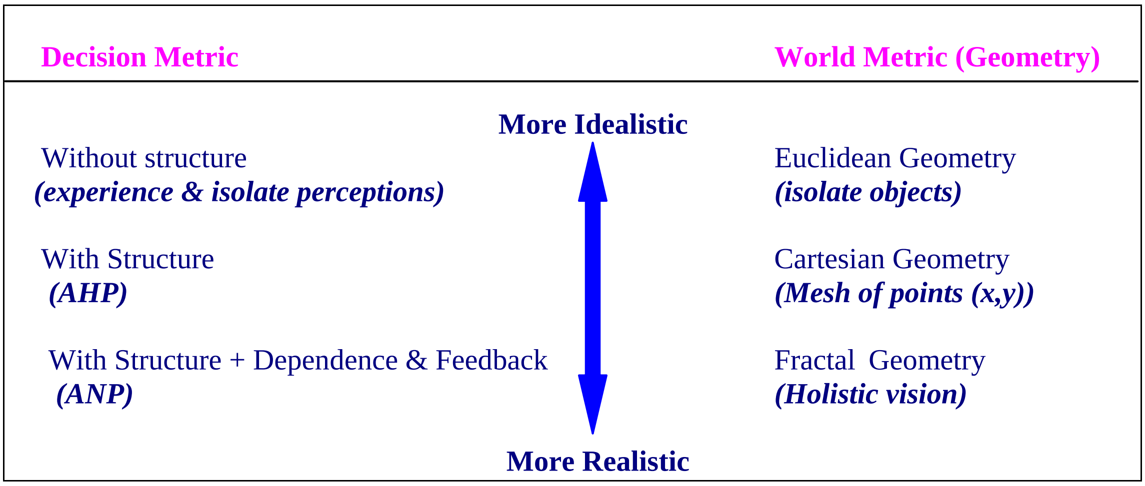


Figure 7: Decision Metric & Metric of the World (Geometry)

Final notes:

As we traffic from an ideal vision of the world towards a more realistic one, the system complexity grows in an overwhelming way, requiring more and more complete and complex systems in order to represent it appropriately. However, the cost of this effort is compensated by our necessity and capacity of building agreements and making better decisions, by our ability to generate models and our capability to predict behaviors in the different branches of the human knowledge, which enables us to continue ahead in our understanding of the world and ourselves.

References:

HALMOS, P. 1964. Measure Theory. D Goes Nostrand Company, Inc. University of Princeton. New Jersey. US. pp 289.

MORRIS KLINE, 1980, Mathematics, The Loss of Certainty. Oxford University Press. US. pp365

OLIVER, D. 1992. Fractal Vision, Put Fractals to Work for You. Sams Publishing. Indiana. US. pp 485.

MATURANA, H. 1997. The Objectivity, An Argument to Force. Dolmen Editions CORP. Santiago. Chile. pp 148.

SAATY, T. 1997. Decision Making for Leaders, The Analytic Hierarchy Process making decisions in a complex world. RWS PUBLICATIONS. Pittsburgh. US. pp 424.

BARNSLEY, M. 1998. Fractal Everywhere (second edition). Academic Press Professional. Masachusetts. US. pp 531.

SAATY, T. 2001 The Analytic Network Process, Decision Making With Dependence and Feedback. RWS PUBLICATIONS. Pittsburgh. US. pp 370.

SAATY, ROZANN W., 2002. Decision Making in Complex Environments: The Analytic Network Process (ANP) for Dependence and Feedback; a manual for the ANP Software "*SuperDecision*"; Creative Decisions Foundation, 4922 Ellsworth Avenue, Pittsburgh, PA 15213.