A SOFTWARE OF ANALYTIC HIERARCHY PROCESS

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ABSTRACT

This paper gives a brief introduction of the software of Analytic Hierarchy Process AHP-NK worked out by the authors.

1. INTRODUCTION

In recent years, the method of Analytic Hierarchy Process proposed by T.L.Saaty has been applied extensively to the fields of society economy decision. The software AHP-NK provides a practical tool of using the method for user. The basic principle of the software is due to T.L.Saaty[1], and also includes some of our results.

2. MODELS

For a single criteria priority, the software provides the three of the fotlwing methods,

(Ⅰ) The right eigenvector method.

(Ⅱ) The least square method.

(Ⅲ) The least deviation method.

A user can choose any number of the methods. In using any one of the above three methods, the judgment matrix can be incomplete.

The least deviation method is proposed by[2]. We make a brief explanation for it here. Let $A=[a_{ij}]$ be a given $n\times n$ judgment matrix and D denote the set consisting of all prority vectors, i.e.

$$D=\{\ w=(w_1, w_2, \cdots, w_n),\ w_i>0,\ \sum w_i=1\ \} \qquad (1)$$

We define a fit deviation between a priority vector w and the fitted matrix A by

$$F(w) = \sum (a_{ij}w_j/w_i + a_{ji}w_i/w_j - 2) \qquad (2)$$

where the summation is taken over all $a_{ij}>0$.

Take the minimal point $w^*=(w_1^*, w_2^*, \cdots, w_n^*)$ of the function F(w) in the set D as a priority vector defermined by the judgment matrix A. [2] has given a detailed discussion for the rationality, theory, and algorithm of this method. We have proved that the minimal point w^* exists uniquely and it is the unique solution in the set D of the equations

$$\sum_{j=1}^{n} a_{ij}w_j/w_i = \sum_{j=1}^{n} a_{ji}w_i/w_j, \quad i=1,2,\cdots,n. \qquad (3)$$

To find the minimal point w^*, In the software we adopt the following iterative algorithm.

ALGORITHM I

1. Take arbitrarily a vector $w(1)=(w_1(1), w_2(1), \cdots, w_n(1)) \in D$ and let k=1.

2. For all i compute

$$r_i(k)= \sum_{j=1}^{n} a_{ij}w_j(k)/w_i(k)$$

and

$$s_i(k) = \sum_{j=1}^{n} a_{ji}w_i(k)/w_j(k).$$

For all i, if $r_i(k)=s_i(k)$, then the computation ends, otherwise truns to step 3.

3. Firstly, choose m such that

$$|r_m(k)-s_m(k)| = \max_i \{|r_i(k)-s_i(k)|\}.$$

Next, let

$$t(k)=[(s_m(k)-1)/(r_m(k)-1)]^{0.4},$$

$$\bar{x}_i = \begin{cases} t(k)w_m(k), & i=m, \\ w_i(k), & i\neq m, \end{cases}$$

and

$$w_i(k+1)=\bar{x}_i/\left(\sum_{j=1}^{n} \bar{x}_j\right), \quad i=1,2,\cdots,n.$$

4. Let k=k+1, then go to step 2.

For the algorithm, we have proved its convergence; if the computation ends at k-th iteration, then the w(k) is the minimal point w^*, otherwise, we have

$$F(w(k+1))<F(w(k)), \text{ for all } k \tag{4}$$

and obtain

$$\lim_{k\to\infty} w(k) = w^* \tag{5}$$

Our experiment for the computing has shown that the computation amount of this method is roughly the same as the eigenvetor method.

For the three priority methods in the software, if the consistency degree of the judgement matrix is not satisfactory, then its elements needs to be revised. In this case, the software will output a prediction matrix $P=[p_{ij}]$, which provides a reference for user. The meaning of the matrix P is explained as follows.

For the eigenvetor mathod, let $w=(w_1,\cdots, w_n)$ and $u=(u_1,\cdots, u_n)$ be the right and left principal eigenvector respectively and the p_{ij} is defined by

$$p_{ij}=(w_i u_j / w_j u_i)^{1/n} \tag{6}$$

For the least square mathod and the least deviation method, we assume that $w=(w_1,\cdots,w_n)$ is the minimal point and then the p_{ij} is defined by

$$p_{ij}=w_i / w_j \tag{7}$$

In [2] and [3] we have proved that, when $p_{ij}<a_{ij}$, if the element a_{ij} is replaced by any value in the interval $[p_{ij}, a_{ij})$ and the element a_{ji} revised correspondently, then the consistency degree of the judgement matrix will gain a improvement.

A usual model of Analytic Hierarchy Process can be partitioned as three types, standard, cycle, and feedback. using the point of view of graph theory, we regard a structure of Analytic Hierarchy Process as a directed greph, where the directed arc expresses a subordination relationship between the components.

A structure type is determined completely by the subordination retationship between the components. speaking precisely, by analysing the adjacency matrix and reachability matrix of a directed graph, we can determine the type of structure. [4] gave algorithms for it. Moreover, having utilized the limit theory of subpermatrix (stochastic matrix), [4] obtained a simple calculation formula for a global composite priority or an impact priority. The software has used these results so that it can adapt to saveral sturcture types and decrease an amount of computing.

3. USAGE OF THE SOFTWARE

The software AHP-NK is written in BASIC Language and runs on IBM-PC. It adopts the way of "conversation" for inputting data and prompts information needed

in Chinese on screen . In the process of inputting data, it posesses some measures of protection so that the user can correct mistakes promptly. The inputted data can be saved as a file on a floppy disk for user's using again after readed. The user can revise the data file. The input data required include component names , subordination retationship between the components, and judgment matries, where the judgment matries can be incomplete. The determining of structure type and the partitioning of hierarchy are carried out automaticaly by the software without usre's inputting any additional information. The software does not restrict the size of a model such as the number of hierachy and the number of component.

The output results of the software include adjacency matries, judgment matries, tocal priority vectors , consistency indexes and global composite priority vectors (or impact priority vectors) of a structure model. If the consistency of the judgment matrix is not good, then it outputs also the prediction matrix P.

REFERNCE

[1] T.L.Saaty, The Analytic Hierarchy Process, Mcgraw-Hill, New York, 1980

[2] Chen Baoqian, Two New Priority Method in AHP, (to appear)

[3] Chen Baoqian, Liou Gueru and Chai Qiaozhu, The Revision of the Judgement Matrix, (a paper contributed in this symposium)

[4] Chen Baoqian, Chai Qiaozhu and Liou Gueru, On Algorithms of Composite or Impact Priority in AHP (to appear)