DYNAMIC JUDGMENTS THEORY ON AHP AND ITS APPLICATION

Wei Yi—Xiang
(School of Management, Beijing University of Aeronautics and Astronautics)
Song Dan-Feng
(School of Management, Beijing University of Aeronautics and Astronautics)

Wang Xiao Yuan
(Aeronautics Science Foundation, China)

ABSTRACT

This paper discusses the key problem of dynamic judgments, Which is the consistency of dynamic matrix. The sufficient and necessary conditions to assess dynamic judgment matrix are developed and the solution form are obtained under the whole consistency condition. According to the steps of solving the dynamic judgment matrix in this paper, judgment and solution to whole consistency problem may be greatly simplified. The approach has been applied to choosing the type of real airplane in China as a concrete instance.
Key words dynamic judgment, consistency of dynamic matrix, whole consistency condition

INTRODUCTION

When we deal with the real problems using the analytic hierarchy process, it is common for us to introduce a function of time into the model. At this time, the judgment matrix become dynamic, whose solution to the eigenvalue problem $A(t)W(t)=\lambda max(t)W(t)$ are weighted vector. However, because the elements in dynamic judgment matrix become the function of time, it is very difficulty to yield the weighted vector. When the dynamic judgment matrix is whole consistency, it is relative simple to obtain the solution. But, the task to judge dynamic judgment matrix whether is whole consistency is another difficult problem. This paper provides a brief method of judging whole consistency of dynamic judgment matrix through theory presented, and the general solutions are also provided.

THEORY

1. The Consistency Theory of Dynamic Judgment Matrix

Definition 1: For the dynamic judgment matrix $A(t)=a_{ij}(t)$, if equation

$$a_{ij}(t) = \frac{a_{ik}(t)}{a_{jk}(t)} \qquad i, j, k = 1, 2, \cdots, n$$

are satisfied for $t_0 \leqslant t \leqslant t_1$, we call A(t) is whole consistency between t_0 and t_1.

Theorem 1: n—order dynamic judgment matrix A(t) is whole consistency if and only if $\lambda_{max}(t)=n$ for $[t_0, t_1]$.

2. The Sufficient and Necessary Conditions of Dynamic Matrix with Whole Consistency

Theorem 2: In the condition of whole consistency, the dynamic judgment matrix A(t) for $[t_0, t_1]$ is

$$X = (1, 1/a_{12}(t), \cdots, 1/a_{11}(t))^T,$$

Where,

$$A(t) = \begin{bmatrix} 1 & a_{12}(t) & a_{13}(t) & \cdots & a_{1n}(t) \\ 1/a_{12}(t) & 1 & a_{23}(t) & \cdots & a_{2n}(t) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1/a_{1n}(t) & 1/a_{2n}(t) & 1/a_{3n}(t) & \cdots & 1 \end{bmatrix},$$

$t_0 \leqslant t \leqslant t_1$, $a_{ij}(t) \geqslant 0$.

Prove: We assume that the eigenvector A(t) is X, namely,

$$A(t)X - \lambda_{max}(t)X = 0$$

Based on the theory 1, we get $\lambda_{max}(t)=n$, substituting into above equation yields

$$A(t)X - nX = 0, \text{ i.e.}$$

$$A(t) = \begin{bmatrix} 1-n & a_{12}(t) & a_{13}(t) & \cdots & a_{1n}(t) \\ 1/a_{12}(t) & 1-n & a_{23}(t) & \cdots & a_{2n}(t) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1/a_{1n}(t) & 1/a_{2n}(t) & 1/a_{3n}(t) & \cdots & 1-n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \\ x_n \end{bmatrix} = 0$$

we obtain,

$$(1-n)x_1 + a_{1n}(t)x_2 + \cdots + a_{1n}(t)x_n = 0 \qquad (1)$$

$$1/a_{12}(t)x_1 + (1-n)x_2 + \cdots + a_{2n}(t)x_n = 0 \qquad (2)$$

$$1/a_{13}(t)x_1 + 1/a_{23}(t)x_2 + \cdots + a_{3n}(t)x_n = 0 \qquad (3)$$

$$\cdots\cdots$$

$$1/a_{1n}(t)x_1 + 1/a_{2n}(t)x_2 + \cdots + (1-n)x_n = 0 \qquad (4)$$

$(1)-(2)\times a_{12}(t)$,

$$nx_1 + na_{12}(t)x_2 + [a_{13}(t) - a_{23}(t)a_{12}(t)]x_3 + \cdots +$$