

**DETERMINING WATER ALLOCATION DURING DROUGHT:  
AN EXAMPLE FROM THE HAWAIIAN ISLAND OF MAUI**

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**1. Introduction**

At any given time, the available water resource is distributed among uses and regions in a well-defined pattern. When supplies are sufficient to meet all demands, conflict over water use is relatively subdued and decisions on water allocation are correspondingly simple. When available supplies are insufficient to meet demands, however, competition for water may grow and decisions on allocation can become complex.

This paper treats the question of how to determine allocation of water during drought and of the role statistical information on meteorological drought can play in such a determination. First, the concept of allocation is discussed, four of its principal elements are identified, and the potential of different kinds of drought to effect changes in an existing allocation is highlighted. Following this comes the description of a model that utilizes information on patterns of past drought to help determine how existing water allocation should change in the face of current or anticipated drought. A worked example of the procedure comprises the last section.

**2. Drought and Water Allocation**

In this paper, allocation of water shall refer to the deliberate distribution of water by use and region, where decisions on the distribution are taken by governmental authorities. Such allocation should consider, at a minimum: (1) the sources of supply and the amounts of water to be provided by each; (2) the demands ("deficits" or "needs") of each use, where identical use classes may be distinguished by region; (3) the costs of supplying the demands; and (4) the benefits of supplying the demands. "Costs" and "benefits" are used here in the broadest possible sense, meaning the full array of positive and negative consequences associated with the allocation.

Drought has the potential to affect some or all of these four elements and hence the allocation itself. For example, during drought demand commonly rises (e.g., irrigation requirements) and supplies fall (e.g., stream flow and reservoirs). Indeed, such effects are implicit in the definitions of the four principal types of drought, a distinction of more than passing importance to the task at hand. Meteorological drought refers to departures from "typical" or "normal" climatological conditions leading to drier than "normal" weather. Much of the difficulty in making this conceptual definition operational lies in the meaning of "normal" and "typical" and the precision given it. Agricultural drought refers to dryness as it affects crops and other plants of importance to agriculture and

livestock. Hydrological drought refers to the insufficient availability of surface and ground waters to meet the demands placed upon them. Finally, socioeconomic drought occurs if social and economic disruptions result, directly or indirectly, when available water is unable to supply demands.

These distinctions are crucial in the recognition of drought as well as in the assessment of the supplies, demands, and consequences which correspond to any given allocation and which thus influence the determination of a preferred one. The public at large and water managers alike commonly respond not so much to meteorological trends as to the effects that such trends have on society. Of particular relevance are agricultural conditions and such hydrologic indicators as stream flow and aquifer levels. Nevertheless, drought-related data available to water managers is commonly limited to studies of meteorological drought--statistical analysis of short- and long-term climatic patterns--with the result that such information may see little direct utilization in drought-management decisions. Yet, as illustrated in the procedure presented below, when linked to the other dimensions of drought climatic data can become a useful decision attribute.

### 3. An Allocation Model

At its most rudimentary, allocation requires the distribution of water from different types and locations of supply to various types and locations of demand (or need). Since drought is apt to affect both supplies and demands, the problem of allocation under drought is conceptually that of deciding the best way to alter pre-drought water distribution such that impacts on supplies and demands are taken into account. Figure 1 illustrates this concept by depicting a hypothetical allocation before a drought (Fig. 1(a)) and the effects the drought has on supplies and demands (Fig. 1(b)).

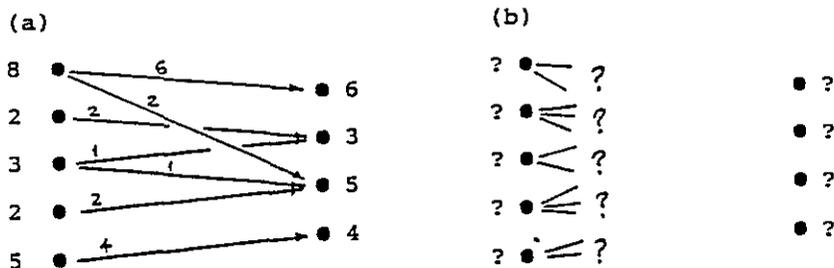


Figure 1. Allocation of water during drought

In this example, available supplies are insufficient to meet demands. This is what we would expect, since in all but the meteorological type some form of deficit is the indicator of drought. Consistent with this is the observation that "available" usually refers to desirable rather than physical limits: just as withdrawing funds from capital reserves does not prevent a business whose expenditures exceed income from going in the red, so a town's pumping water from an aquifer beyond its sustainable yield does not eliminate the deficit. This distinction becomes important when it is deemed desirable (perhaps only in the short run) to overallocate available supplies.

Two major questions arise. First, how can we estimate a drought's effects on demands and supplies? Since lead time may be desirable, even required, in order to change an allocation, this question includes the problem of how to predict a drought. Also of interest here is the way in which statistical analysis of meteorological drought might aid in such prediction. Second, given the predicted effects upon available supplies and demands, how should a new allocation be determined? The sections below discuss a set of procedures to follow in answering these questions.

**Estimating Drought Impacts.** How a drought affects water supplies and demands depends on the location of the supplies and demands and on the severity of the drought. One way to estimate such overall effects would be to link direct and indirect effects through a cause-effect chain. One could then determine the relationship defining each link of this chain and, by appropriately combining them all, assess the ultimate consequences. Clearly, care is required so that the approach does not become overly reductionist and misrepresent, or miss altogether, important systemic characteristics of the set of individual relationships considered as a whole.

In the present context, three observations become vitally important. First, there may be no way, practicable or otherwise, to measure any of these drought-induced consequences in an objective manner. This means that people's judgments will be important and will need to be incorporated into the assessments. Related to this is the fact that effects will be felt upon more than one element of each relevant impact class (such as regions, societal sectors, water sources, and supply systems), and it will be useful to know how a given drought consequence affects one element as compared to another. Thus, relative impacts are important. Third, quantitative (based on ratio-scale data) rather than qualitative assessments will be more useful in determining water allocation since allotments as percentages of the total available supply are what is sought. The Analytic Hierarchy Process (AHP) offers an approach to estimating and evaluating such impacts that responds to these three desiderata (Saaty 1980). Because during the last decade the AHP has received considerable attention from

decision scientists and practitioners alike, the remainder of this paper discusses its application to water allocation during drought rather than the methodology per se.

**Drought Impacts as an Analytic Hierarchy.** Hierarchical structures can be used to represent the effects upon water supplies and demands during and after a drought. Supply effects can be depicted for each source region through a four-level hierarchy. The apex of the hierarchy (Level 0, or L(0)) represents the overall goal of determining how drought is apt to affect water supplies. Immediately below it, at L(1), would be different drought scenarios. These scenarios would distinguish droughts of different magnitude and embody characteristics meaningful to water-resource managers, such as duration and degree of dryness. Level 2 would show the effects of droughts of different severity on the input of water from the natural hydrological system to the supply system. Such effects could be represented by quantitative estimates, expressed as ranges, of the degree to which the pre-drought input might be altered under a given climatic scenario. In turn, level 3, would depict the effects of those changes upon the final supply availability. The demand hierarchy corresponding to each demand area would consist of analogous levels. Level 1 would represent the climate scenarios, L(2) the sectors or uses (e.g., agriculture) likely to be affected, and L(3) the quantitative estimates of the relative changes in water demand by the sectors above.

Following AHP convention, the elements at each level in the hierarchy would be prioritized by comparing them pairwise with respect to relevant elements at the next higher level (Saaty 1980). At L(1) of a given supply hierarchy, for example, we would ask, "How much more likely is climatic scenario i than scenario j?" If groundwater is an important source, the assessment question at L(2) might be: "Under scenario i, how much more (less) likely is it that infiltration would be reduced by 5% to 10% than from 10% to 15%?" Finally, the L(3) elements would be compared thus: "Given that scenario i results in infiltration's declining by 10% to 15%, how much more likely is the reduction in sustainable yield (relative to a given hydraulic head) to be between 0% and 5% than between 5% and 10%?" Summing these final priorities yields the area's estimated percentage change in supply for the planning period.

The queries pertaining to each demand hierarchy are somewhat different. After comparing the scenarios at L(1) with respect to likelihood, sector m is compared to sector n at L(2) according to the amount of water consumed by each under non-drought conditions. Since the assessments are made relative to only one scenario, they will be identical for all reference scenarios at L(1). If use data are available, direct assessments may be used; otherwise, one asks, "How much more (less) water is (typically) consumed by sector m

than sector n under non-drought conditions for this time of year?" The result is a weight for each sector in proportion to its "normal" (non-drought) water usage. In contrast to the comparisons at L(2), those at L(3), assessing the relative likelihood of each demand-modification factor, do distinguish among climatic scenarios: "Under climatic scenario i, how much more (less) likely is it that sector m's demand will rise between 5% and 10% than between 10% and 15%?" Summing these final priorities yields the estimated percentage change in the study area's total demand. Multiplying this demand-modification factor by the non-drought use gives the area's new demand for the target period, corresponding to a demand node in Fig. 1(b).

**Assessing Drought Likelihood.** Drought scenarios appear at level 2 in the demand and supply hierarchies discussed above, and the statistical characterization of drought can be used to aid the assessment of the likelihood of such scenarios. Two tasks are required, the specification of a scenario and the estimate of its probability.

Drought scenarios are defined by first specifying a period of interest and then a small number of values of a selected drought attribute. Given k such values, k+1 scenarios will be defined, each scenario corresponding to a drought condition falling between two adjacent values. For example, at the end of June a water manager might be interested in the likelihood of drought in July and the consequent increased demand for irrigation water. If a minimum of 30 mm of rain were required during July in order to avoid losses to the crop in question, the manager could specify precipitation (P) as the drought attribute and one meaningful value equal to 30 mm, *i.e.*  $P_1 = 30$ . With that single value, two scenarios would be defined, one with rainfall less than 30 mm and the other with 30 mm or more. If another value were also specified, such that  $P_2 = 20$ , then three scenarios would be defined: when  $P \leq 20$ , when  $20 < P \leq 30$ , and when  $P > 30$ . Although in this example the attribute is precipitation, many others are possible; a drought index, such as the Palmer Drought Severity Index, would be one. Likewise, one may prefer to specify rainfall amounts in terms of return periods rather than depth in millimeters.

Once scenarios are defined and specified, two basic approaches to estimating their probabilities may be employed. One way calculates the probabilities of each scenario in the future period of interest based on the frequency of that condition during the period of record. For example, consider once again that July is the period of interest, that precipitation amount  $P_1$  is the chosen attribute value, and that n is the number of consecutive years of precipitation record. Then if  $P_1$  has been exceeded m times during the period of record, the probability that  $P_1$

will be exceeded in July can be taken as  $m/n$ ; that is,  $\Pr(P_{\text{July}} > P_1) = m/n$ . (Hydrologists usually modify this formula slightly to obtain a "plotting position," but the concept remains the same.) This approach is simple to use, but it ignores the history of the current drought, since it assumes that the probability of exceeding or falling below the attribute (in this example, precipitation) value in July is independent of the values obtained for that attribute in the immediately preceding months.

If one believes that an attribute's value for a given period of interest depends significantly upon such values for previous periods, then one should incorporate available information on those values as well. Guidance on how to do this comes from Bayes' Theorem:

$$\Pr(AB) = \Pr(A|B)\Pr(B) = \Pr(B|A)\Pr(A).$$

Continuing with the same example, let event A be July's receiving precipitation of  $P_1$  or greater. Similarly, let event B refer to the amount of precipitation received in some period of interest prior to July, say June. More precisely, let B represent that period's (June's) receiving an amount of precipitation equal to or less than  $P_0$ . Assuming that A is dependent on B, one would like to estimate the joint probability of the two events,  $\Pr(AB)$ . Since B has already occurred, its probability is 100% and  $\Pr(B) = 1.0$ . Thus, all that is needed is an estimate of the conditional probability  $\Pr(A|B)$ .

To estimate  $\Pr(A|B)$ , one first identifies the years of record in which the corresponding "preceding period" (*e.g.*, June) received precipitation of  $P_0$  or less. Suppose there are  $j$  such years,  $j \leq n$ . One then determines how many of those years registered precipitation of  $P_1$  or more. If there were  $i$  such years, then the conditional probability of getting precipitation at least equal to  $P_1$  is  $i/j$ ; for this example,  $\Pr(A|B) = \Pr(P_{\text{July}} \geq P_1 \mid P_{\text{June}} \leq P_0) = i/j$ .

The decision to use simple probabilities implies the belief that the future period of interest is independent of previous periods. The use of conditional probabilities implies those events are dependent. These mark the two ends of the continuum, since the less the independence the closer to the "conditional" end the true probability would lie, and vice versa. But one does not know with certainty what the degree of dependence is, and it will vary with the attribute used, the region, and the months of interest. Therefore, in estimating the likelihood of a given scenario, the water manager may specify a probability different from either of these yet based on (*i.e.* informed by) both. The importance s/he gives to the conditional probability reflects the degree of persistence s/he feels is present in the index used.

**Determining a New Allocation.** Given the estimated effects of drought on supplies and demands, how should the current allocation of water be modified? Assuming the social acceptability (if not optimality) of the existing allocation, and that some cut in demand is necessitated (or merely desirable--e.g., for aquifer management), a common approach is simply to spread any necessary reduction evenly, in percentage terms, across all users. Such a "proportional rollback," however, does not consider the distribution of drought impacts, either upon supplies or upon demands. Modifying the supplies and demands by the factors determined by the AHP procedure just described, however, does indeed consider such impacts. If one now wishes to reallocate the resource in an optimal manner, considering these anticipated changes in supplies and demands, a constrained optimization model such as the following may be employed.

Let  $x_{ijk}$  represent the amount of water in millions of gallons per day (mgd) that water-supply system  $j$  will get from source  $i$  and provide to user  $k$ . Also, denote by  $S_i$  the available supply (mgd) at source  $i$ , and by  $D_k$  the demand (mgd) by user  $k$ . In addition, let  $C_{ij}$  be the transfer capacity (mgd) between source  $i$  and system  $j$ , and  $C_{jk}$  the transfer capacity between system  $j$  and user  $k$ . Then in times of shortage any allocation must meet the following conditions:

1) Water provided to some supply system cannot exceed source capacity:

$$\sum_j x_{ij} \leq S_i \quad \text{for all } i, i = 1, 2, \dots, n \quad [1]$$

2) Water entering system  $j$  from source  $i$  either supplies users  $k$  or is stored within system  $j$ :

$$\sum_i x_{ij} - \sum_k x_{jk} \geq 0 \quad \text{for all } j, j = 1, 2, \dots, m \quad [2]$$

3) Water transfer between sources and supply systems cannot exceed limits on transfer rate:

$$x_{ij} \leq C_{ij} \quad \text{for all } (i, j) \text{ links} \quad [3]$$

4) Water transfer by the supply system to users must not exceed system limits on the rate of such transfer:

$$x_{jk} \leq C_{jk} \quad \text{for all } (j, k) \text{ links} \quad [4]$$

5) Determine the deficit  $d_k$  between user  $k$ 's demand and the amount received:

$$\sum_j x_{jk} + d_k = D_k \quad \text{for all } k, k = 1, 2, \dots, s \quad [5]$$

Except for a slight variation in [5], the above constraints are those comprising the well-known transshipment problem in linear programming. [5] differs from the standard formulation in that, due to the supply shortage, users' demands are not required to be met.

In the transshipment problem, the objective is usually to minimize the total cost of the distribution. Here, we can think of minimizing at least two different costs. One refers to the monetary (financial) cost associated with the physical transfer of the water. Letting  $c_{ij}$  denote the cost of moving 1 mgd between source  $i$  and supply system  $j$ , and  $c_{jk}$  that between the supply system and user  $k$ , the objective function would be to minimize COST:

$$\sum_i \sum_j c_{ij} x_{ij} + \sum_j \sum_k c_{jk} x_{jk} - \text{COST} = 0 \quad [6]$$

Another cost is that incurred by society at large, including that corresponding to the individual user, when supplies fall short of demands. Hence, another objective is to minimize DEFICITS, the sum of weighted deficits:

$$\sum_k w_k d_k - \text{DEFICITS} = 0 \quad [7]$$

The weights  $w_k$  signify that a unit shortfall from one user's demand does not necessarily represent the same cost, or importance, to society (or to that user) as does a similar shortfall from another user's demand. Weights can thus be assigned to reflect these different costs if so desired.

#### 4. An Illustrative Example

To illustrate the overall procedure, let us consider a simplified example patterned after and reflecting in a general sense the situation found on the Hawaiian island of Maui. Since some of the data used here are hypothetical, the quantitative dimension should be viewed as illustrative only.

**Drought's Impacts on Supply and Demand.** The first step is to structure analytic hierarchies to estimate drought effects upon water supplies and demands. Figure 2 shows a hierarchy corresponding to changes in available water supply in one source area, that of the Iao System.

The month of April was selected as the period of interest, and four drought scenarios were defined by return period: an "extreme drought" is one that would occur no more often, on average, than once in 20 years; a "bad drought" corresponds to one more frequent than an extreme drought but

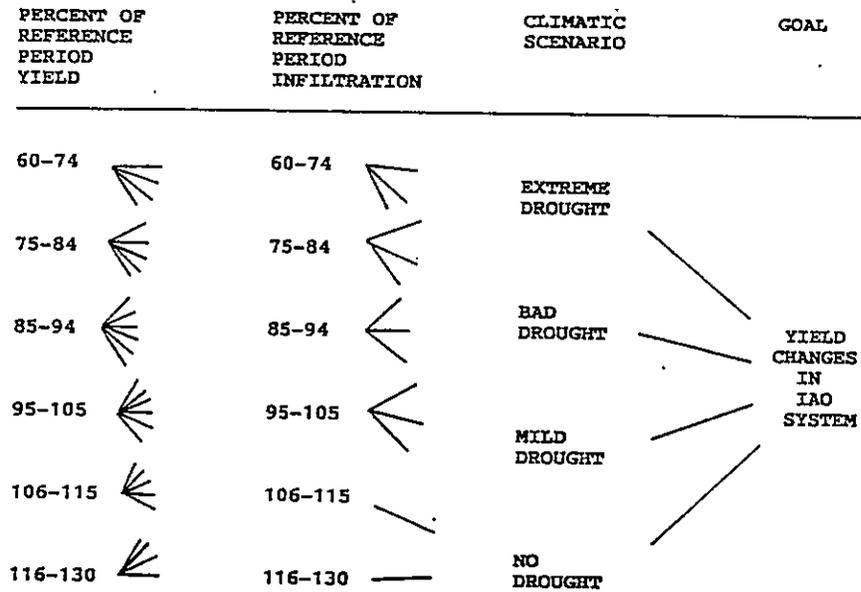


Figure 2. Hierarchy to estimate changes in water yield

still likely to occur no more often than once in 10 years; a "mild drought" has an expected return frequency not exceeding once every 5 years but more often than the more severe droughts; and "no drought" refers to all other cases (Giambelluca et al. 1990). Probabilities of these scenarios are then, respectively, 5%, 5%, 10%, and 80%.

Infiltration-modification factors covered intervals ranging from 0.60-0.74 to 1.16-1.30. Higher factors were included in the comparisons under the "no drought" scenario, while lower ones were compared for more severe droughts. Finally, the yield-modification factors chosen for the evaluation ranged from 0.60 to 1.30. Table 1 shows the final ("global") priority estimated for each yield-factor interval.

Table 1  
Likelihood Weights for Yield-Modification Factors for the Iao Water Source

Factor Interval	Mid-point	Likelihood Weight
0.95-1.05	1.00	0.242
1.06-1.15	1.10	0.224
0.85-0.94	0.90	0.173
1.16-1.30	1.23	0.155
0.75-0.84	0.80	0.127
0.60-0.74	0.67	0.079

The hierarchy corresponding to changes in water demand in the Wailuku-Kahului Community Plan Area is shown in Figure 3. The drought scenarios in level 1 were defined as

in the supply hierarchy, but since this area is not coincident with that comprising the Iao System, the actual precipitation amounts to which they refer are different. Six different uses are distinguished at level 2: interior and exterior uses for each of the domestic (residential), commercial (including tourist facilities and resorts), and public sectors. In this example, agricultural uses were omitted since the focus is on municipal water allocation.

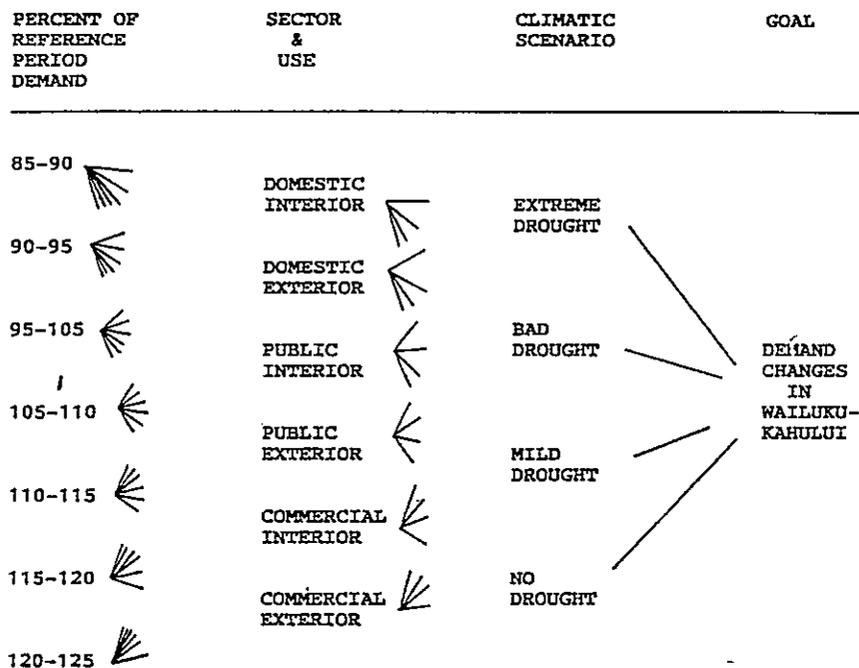


Figure 3. Hierarchy to estimate change in water demand

Seven demand-modification factor intervals comprise the alternatives at level 3. Table 2 indicates their global relative weights.

Table 2  
Likelihood Weights for Demand-Modification Factors for the Wailuku-Kahului Community Plan Area.

Factor Interval	Mid-point	Likelihood Weight
0.95-1.05	1.000	0.345
1.05-1.10	1.075	0.212
0.90-0.95	0.925	0.169
1.10-1.15	1.125	0.150
0.85-0.90	0.875	0.112
1.15-1.20	1.175	0.008
1.20-1.25	1.225	0.005

Figures 2 and 3 illustrate the hierarchies for only 1 supply and 1 demand area respectively. On Maui, there are 25 such "systems" comprising the 6 source sectors in the groundwater classification currently being followed. Similarly, Wailuku-Kahului is only 1 of 6 Community Plan Areas. Each of these would require its own hierarchy and associated assessments.

**The Water Allocation Model.** To illustrate how the prioritized yield and demand-modification factors can be used to help determine the "best" water allocation at the onset of a drought period, consider a situation in which 8 source areas must supply 6 demand regions. Table 3 shows each source's available supplies (mgd) at the end of March, the consumption of the demand areas at that time, and the sources capable of supplying each demand region. With all variables in units of mgd, and assuming no intermediate water-supply systems and the single objective of minimizing equally-weighted deficits, the standard transportation (rather than transshipment) model can be used. The optimal allocation allows all deficits to be met while leaving excess capacity at the IAO, UKUMEHA, and KIPAHULU sources (Table 4).

**Table 3**

Sources, Demand Regions, Supplies, Demands, and Source-Demand Links for the Pre-Drought Allocation Situation.

Supply (mgd)	Source Name	Source Number	Possible Demand Regions					
			A	B	C	D	E	F
13.11	IAO	1	*	*		*		
2.20	WAIHEE	2	*	*				
6.00	UKUMEHA	3		*	*			
2.80	LAUNIU	4			*			
1.10	MAKAWAO	5				*	*	
3.85	HONOPOU	6				*	*	*
1.0	KIPAHULU	7				*	*	
0.07	KEANE	8				*		*
Demands (mgd):			7.66	7.08	8.76	0.95	4.77	0.28

NB: A = WAILUKU; B = KIHEI; C = LAHAINA; D = PAIA;  
E = KULA; F = HANA

Now the modification factors determined via the AHP come into play. Using the midpoint of each modification-factor interval, multiplying it by the weight of that interval, and summing the products, one obtains the weighted-average yield-modification factor for the IAO source area; in this case, it is 0.989. Multiplying this by the end-of-March capacity for IAO, 13.11 mgd, one gets 12.97 mgd, the estimated availability for April. By a similar procedure, the weighted average for the Wailuku-Kahului Community Plan Area is 1.012, which, when multiplied by the

end-of-March consumption figure for that region (7.66 mgd), yields 7.75 mgd as its projected April demand. Following the identical procedure for all source and demand regions results in new limits and demands (Table 4, column 4).

**Table 4**  
Supplies, Demands, Excess Supplies, and Deficits (in mgd) for the Optimal Allocation under Each Model Examined.

<u>Supplies</u>	<u>Model B</u>		<u>Model I</u>		<u>Model II</u>	
	<u>Limit</u>	<u>Excess</u>	<u>Limit</u>	<u>Excess</u>	<u>Limit</u>	<u>Excess</u>
IAO	13.11	0.57	12.97	0.25	12.97	--
WAIHEE	2.20	--	2.15	--	2.15	2.15
UKUMEHA	6.00	0.04	6.00	--	6.00	--
LAUNIU	2.80	--	2.71	--	2.17	--
MAKAWAO	1.10	--	1.02	--	1.02	1.02
HONOPOU	3.85	--	3.79	--	3.79	--
KIPAHULU	1.00	0.02	1.00	--	1.00	--
KEANE	0.07	--	0.06	--	0.06	--
<u>Demands</u>	<u>Demand</u>	<u>Deficit</u>	<u>Demand</u>	<u>Deficit</u>	<u>Demand</u>	<u>Deficit</u>
WAILUKU	7.66	--	7.75	--	7.75	7.75
KIHEI	7.08	--	7.12	--	7.12	--
LAHAINA	8.76	--	8.83	0.12	8.83	2.83
PAIA	0.95	--	0.99	--	0.99	0.99
KULA	4.77	--	4.91	0.03	4.91	1.12
HANA	0.28	--	0.33	0.33	0.33	0.27

Modifying the supplies and demands in Model B to reflect these estimated changes yields a new model, Model I. The optimal allocation under this model would leave IAO as the sole source with excess supply, and deficits would occur in LAHAINA, KULA, and HANA (Table 4, column 5).

While the single objective in Model I is to minimize equally-weighted deficits, that in Model II attempts to minimize the total cost of water transfer as well. Including cost coefficients (arbitrary, in this case) in [6], putting COST in the objective function alongside DEFICITS, and varying the objective function coefficients, one can now explore the consequences of assigning different priorities to the objectives. When both have coefficients of 1.0, the results are as shown in Table 4 (column 7).

The differences between the solutions to Models I and II demonstrate that the best allocation depends on the objectives being considered and the weight given them. They also point up the importance of how an objective is defined and measured to begin with: there is no a priori reason, for example, why all deficits should be assumed of equal consequence.

## 5. Conclusion

Making decisions regarding the allocation of water under scarcity is often complex and always value-laden. In areas normally blessed with sufficient water to meet demands, the occurrence of drought frequently requires allocation decisions to be made rather hastily, without the benefit of a well-thought-out procedure to guide them. A common practice in such cases is to require across-the-board cuts in consumption which are percentage-wise equivalent. Such a practice is arbitrary and, notwithstanding its "proportional equality," is neither equitable nor efficient.

The approach presented here provides a way out of such arbitrariness while simultaneously revealing the values employed in the allocation decision. Beginning with the supply capacities and demands prior to water shortage, the procedure uses empirical data on the relevant hydrological systems and consumption patterns, together with one's judgment, to estimate changes to supplies and demands which are likely to occur during a future period. The future period is characterized by a set of climatic scenarios whose probabilities may be based in part on the historical record. Once the likely changes are determined, multiobjective optimization is used to identify an allocation which best corresponds to one's view of the relative importance of the objectives and the way in which they are defined.

## References

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