The Analytic Hierarchy Process and Artificial Intelligence: A Comparative Study

Zhu Song-chun
Wang Hong
Institute of Military Operations Research and Analysis
Academy of Military Science of the P.L.A. of China
P.O.Box 998(11), Beijing,100091, P.R.China

Abstract

The AHP, a methodology for solving and analyzing macro and complex decision-making problems, has inherent relationships with rule-based expert systems (ES) which are developed to solve micro problems in some specific fields. In this paper, after comparing the knowledge in decision-making underlying the hierarchies with more detailed knowledge represented in Expert Systems (ES), we formalize these knowledge to IF-THEN rules by extending casual relationships to the relationships of contributions and, by introducing the Certainty Factor (CF) model of MYCIN, to "infer" priorities. The approximate reasoning of the CF model is similar to the synthesis in the AHP, both yield the same rank in a given example but with a little different value of weights or certainty factors. As a direct generalization from incorporation of the AHP and ES, if there are relations of AND or OR among the elements in a hierarchy, the CF model will give out ranks; if detailed explanation is needed, ES will work well. On the other hand, AHP will play an important role in acquiring high level knowledge for "qualitative reasoning." The incorporation of the AHP and ES will benefit both tools in decision-making.

1. Introduction: The AHP and AI in Decision-making

Problems or systems on which the AHP [1,2,3] is used in decision-making are ill-structured complex macro problems. In such problems or systems, the partitions and identifications of elements or subsystems are uncertain or, it seems, that there are many reasonable ways to decompose them for multi-objective decisions.

Meanwhile, people always find it difficult to make satisfactory tradeoffs between reality and clear-cut solutions, especially in the case of dividing a problem into one small enough to find inherent relations and model them mathematically. As a result, rough partitions or categorizations could be thought of as

This research was partially supported by the National 863 Funds for the High Technology Program of China under project #863-306-402.

the first step in solving high-level decision problems. The AHP suggests a systematic way of structuring hierarchies into which people can put their overall considerations. In solving such problems people will use their imagination, feelings, experiences, and judgments, which serve as the basis for deduction and induction, but would not be always consistent to them, i.e., deduction and induction are necessary but not sufficient. Furthermore, to relate and measure a great number of important qualitative elements for deduction and induction which are basic methods to conclude agreeable results, people have to set up measurement to evaluate different observations reflecting the objective laws in some perspectives and revise the decision in practical decision-making processes.

On the other hand, the uncertainty (probability, fuzziness, possibility, etc.) of partitions and judgmental values leads to the uncertainty of conclusions. But in the description level of a problem to be solved, people seem to pay more attention to the degree of difference among a possible set of conclusions or alternatives from which the satisfactory solution is selected.

Research on the Artificial Intelligence (AI) over the last three decades has made progress in many fields. Thousands of expert systems (ES) have been developed today. Those ES perform certain tasks requiring experts' knowledge in a certain domain so that they resemble human intelligence to solve problems. For example, MYCIN [4,5], a rule-base ES, diagnoses blood infections. It informs itself about particular cases by requesting information about a patient's symptoms, general condition, history and the easy, quickly obtained laboratory tests. At each point, the question MYCIN asks is determined by MYCIN's current hypothesis and the answer to all previous questions, but they vary as evidence builds.

Knowledge in ES solving decision problems are characterized by a large amount of obtainable and reliable detailed expertise in a particular domain and, in representing them, they should have relatively obvious casual or other structural relations so that we can perform predicate calculus and/or approximate reasoning to find truthful or probabilistic solutions to the problem at hand. Presumably, the most generally applicable casual descriptions are those stated in terms of the fundamental laws of physics. Attempts to describe problems at this level, however, are generally impractical, not only for high-level decisions, but for ES as well. The objects of concern to decision makers, mathematicians, and, of course, system builders, are at a much higher level of description. But as the levels of description go higher, the casual relation will become more and more vague, leading to a ill-structure. Therefore, knowledge acquisition at such level is particularly hard and of variety.

Just as ES is good for processing deep level expertise, the AHP is suitable for macro qualitative and quantitative analysis. We are motivated to consider the necessity and possibility of incorporation of them to address decision problems at different levels from the very detailed bottom to the ultimate top.

Xu [6] pointed out that the interrelationship between the AHP and AI is worth to be studied. Up to now, the literature on this subject is scarce. In the next section of this paper we will start our comparisons from some detailed perspectives.

2. Comparisons of the AHP and ES

In fact, knowledge underlying hierarchies, relations and qualitative judgments in the AHP have inherent associations with expertise in rule-based ES. The key to link and unify them is the way we represent them instead of changing their original and inherent implications. So, let us consider some definitions and concepts in representing knowledge and in approximate reasoning in MYCIN. Table 2.1. shows some items for comparison. This table suggests:

(1) The AHP is applicable to high level descriptions involving a great number of uncertain factors, while ES is good for detailed and casual descriptions;

(2) Elements in the AHP involve relatively comprehensive and complex concepts, so the casual relations drastically reduced to such relations as dominate, hierarchical level of concepts, etc. As a result, if the answer to the problem can also be described by similar complex concepts, then we might only need to consider all general concepts with uncertainty and their roughly hierarchical relations. In contrast, elements in MYCIN generally involve smaller and simpler concepts for micro descriptions, among which casual relations play an important role. Nevertheless, dominance contribution, and importance in the AHP are a natural extension of intensity of the antecedent supporting the successor in IF-THEN rules in MYCIN. But relations of AND, OR are so vague that they

are included in dominate. Therefore, at this point, IF-THEN rules can be applied to represent relations among elements in high level decision problems as long as there are some measurements.

Table 2.1. Comparisons of the AHP and MYCIN (ES).

	AHP	MYCIN(ES)
element	goal, subgoal, criteria, plan, scenario, weight,...	fact, reason, consequence, conclusion, evidence, hypothesis, CF,...
mono-relation	dominance, feedback, contribution, importance,...	IF E THEN H (CF)
multi-relation	hierarchy, positive reciprocal, matrix,	AND, OR, AND/OR graph, CF model of
	independent additive synthesizing	approximate reasoning
problem	priority, finding weights	diagnose, inference from evidence to conclusion and CF
information needed	structuring hierarchy from top to down, making judgments by pairwise comparison	giving all facts, rules and CFs
measurement setting	relative and/or absolute ratio scale	casual relation, probability, experience
consistency	C.I.	manual or run-time test
cost in solving problem	low	high

(3) Consistency Index (C.I) in the AHP makes the inference of IF-THENs consistent to formal logic so as to reduce conflicts in MYCIN. Manual or run-time test of conflicts may cause some problems. So, the AHP seems to be advantageous to MYCIN in acquiring high level knowledge.

3. Formalized Descriptions of Knowledge for the AHP and ES and Its Approximate Reasoning

Because IF E THEN H(CF) means and represents the intensity or contribution of E supporting H, thus for a hierarchy of AHP in Figure 3.1, we can think of weight w_1, w_2, ..., wn of criteria C_1, C_2,

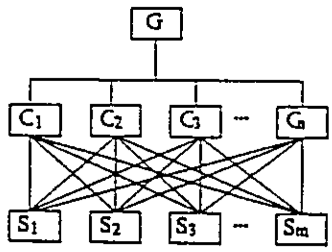


Figure 3.1. A typical hierarchy of AHP

..., C_n with respect to goal G as intensity CF_1, CF_2, ..., CF_n of evidence E_1, E_2, ..., E_n supporting conclusion H, ..., and so on, hence leading to a set of formalized rules

IF C_1 THEN $G(w_1)$,

IF C_2 THEN $G(w_2)$,

......

IF C_n THEN $G(w_n)$;

IF S_1 THEN $C_1(w_{11})$,

IF S_2 THEN $C_1(w_{21})$,

......

IF S_m THEN $C_1(wm_1)$;

IF S_1 THEN $C_2(w_{12})$,

IF S_2 THEN $C_2(w_{22})$,

......

IF S_m THEN $C_2(w_{m2})$,

......

IF S_m THEN $C_n(w_{mn})$,

where, w_{ij} denotes S_1's weight (intensity) with respect to C_1 (conclusion). Then, when we ask IF S_1 THEN G(?) the system we build should infer the same rankings of S_1, S_2, ..., S_m as the AHP's synthesizing algorithm does.

MYCIN defines three concepts: CF (Certainty Factor), MB (Measure of Belief) and MD (Measure of Disbelief). When evidence is a single condition, for the rule IF E THEN H (CF[H,E]), if E exists with certainty, i.e., CF[E]=1, we have:

$$CF[H]=CF[H,E]; \quad (1)$$

If E exists with uncertainty, i.e., CF[E] < 1, we have:

$$CF[H]=CF[H,E]*\max\{0,CF[E]\};$$

Considering for an element in Figure 3.1,

$\forall E$, CF[E] >0,

we have

$$CF[H]=CF[H,E]*CF[E]. \quad (2)$$

When two rules have the same conclusion, i.e., IF E_1 THEN H (CF[H,E_1]) and

IF E_2 THEN H (CF[H,E_2),

we have

$$CF_{12}[H] = \begin{cases} CF_1[H]+CF_2[H]-CF_1[H]*CF_2[H], & \text{if } CF_1[H] \ \& \ CF_2[H] > 0 \\ CF_1[H]+CF_2[H]+CF_1[H]*CF_2[H], & \text{if } CF_1[H] \ \& \ CF_2[H] < 0 \\ CF_1[H]+CF_2[H], & \text{otherwise.} \end{cases}$$

Obviously, for elements in Figure 3.1., we have

$$CF_{12}[H] = CF_1[H]+CF_2[H] - CF_1[H]\ CF_2[H] \qquad (3)$$

By induction we can easily prove that if E_1, E_2. ..., E_n support the same conclusion H in Figure 3.1., we have

$$CF_1[H] = CF[H,E_1]\ CF[E_1] \qquad (4)$$

and

$$CF_{12\ldots n} = \sum_{i=1}^{n} CF_i[H] - \sum_{1<i<j<n} (CF_i[H]\ CF_j[H]) + \sum_{1<i<j<k<n} (CF_i[H]\,CF_j[H]\,CF_k[H]) - \ldots + (-1)^{n-1} \prod_{i=1}^{n} CF_i[H] \qquad (5)$$

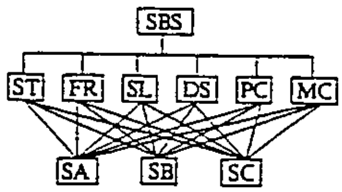
Here, we have found the difference between the AHP's synthesizing algorithm and (5) in that, in the former, independence makes weights to be added, while in the latter, inevitable dependence of evidence makes CF's be added and then subtract the overlap of evidence. The term subtracted is of order higher than

$$\sum_{i=1}^{n} CF_i[H]$$

Therefore, the absolute value of the two methods is slightly different, but the following example in the next section will show that, this difference does not appear to cause rank reversals.

4. Incorporation of the AHP and ES: An Example

The following example is adopted from [1] "selecting a best school". Figure 4.1 gives the hierarchy of the AHP method.



SBS--Selecting a Best School
ST--Study
FR--Friendship
SL--School Life
DS--Discipline
PC--Preparation for Coolege
MC--Music Course
SA--School A
SB--School B
SC--School C

Figure 4.1. Hierarchy of Selecting a Best School

From the judgment matrices given in [1], we have rules:

IF ST THEN SBS (0.32),

IF FR THEN SBS (0.14),

IF SL THEN SBS (0.03),

IF DS THEN SBS (0.13),

IF PC THEN SBS (0.24),

IF MC THEN SBS (0.14),

IF SA THEN ST (0.16),

IF SA THEN FR (0.33),

IF SA THEN SL (0.45),

IF SA THEN DS (0.77),

IF SA THEN PC (0.25),

IF SA THEN MC (0.69).

Then, when we ask: IF SA THEN SBS(?), we have a reasoning graph like Figure 4.2.

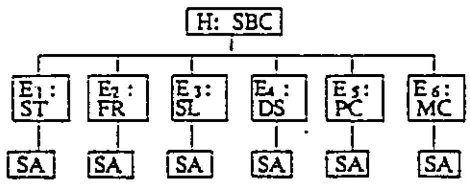


Figure 4.2. Reasoning graph: a special case of AND-OR tree.

From the AHP we have

$CF\ [SA] = 1$, $CF\ [E_1] = 0.16$, $CF\ [E_2] = 0.33$,

$CF\ [E_3] = 0.45$, $CF\ [E_4] = 0.77$, $CF\ [E_5] = 0.25$,

$CF\ [E_6] = 0.69$;

By (4), we have

$CF_1\ [H] = CF\ [H, E_1]\ CF\ [E_1] = 0.16*0.32 = 0.0512$,

$CF_2\ [H] = 0.0462$, $CF_3\ [H] = 0.0135$,

$CF_4\ [H] = 0.1001$, $CF_5\ [H] = 0.06$,

$CF_6\ [H] = 0.0966$.

By (5), we have

$$CF_{123456} = \sum_{i=1}^{6} CF_i[H] - \sum_{1<i<j<n} (CF_i[H]\ CF_j[H]) + \sum_{1<i<j<k<6} (CF_i[H]\, CF_j[H]\, CF_k[H]) - \ldots + (-1)^5 \prod_{i=1}^{6} CF_i[H] = 0.32 \tag{6}$$

Similarly, when we ask: IF SB THEN SBS(?), we have:

$CF_{123456}\ [H]_2 = 0.334,$ (7)

and when we ask: IF SC THEN SBS(?), we have:

$CF_{123456}\ [H]_3 = 0.229.$ (8)

[1] yields the following priorities:

$$[0.37,\ 0.38,\ 0.25]^T.$$

On the other hand, if we normalize (6),(7), and (8), we have

$$[0.362,\ 0.378,\ 0.259]^T.$$

So we have the same rank in spite of slight difference in values.

5. Conclusions

The endeavor of this research is trying to find a formalized representation and the inference method by incorporating the AHP with ES for both macro and micro decisions. The method explored above is only a simple, direct (therefore, primitive) extension of ES to the AHP as the example shows. But we could conclude, from the point of view of supplementation and support by each other, that

(1) There is a unified and formalized representation of knowledge, rule-based production system can represent high-level knowledge;

(2) If the judgment needs explanation by more detailed rules, we could call back related rules used by inference (it depends on the granularity of rules in ES);

(3) Consistency Index makes high level knowledge more logically consistent, such that it provides us with a new tool;

(4) Relations like AND and OR in ES can be introduced into the

hierarchy of the AHP when necessary, and this could be think of as an extension to the AHP;

(5) the approximate reasoning in MYCIN gives the same rank as the synthesizing algorithm of the AHP does;

(6) The AHP provides another way to acquire high level knowledge for decision-making.

Since space is limited, further conclusions will be discussed separately.

REFERENCES

1. T.L.Saaty, The Analytic Hierarchy Process, McGraw-Hill 78International Book Company, N.Y. 1980.
2. T.L.Saaty, Decision Making for Leaders, Lifetime Learning Publications, Belmont, California, 1982.
3. T.L.Saaty & J.M.Alexander, Thinking With Models, Pergamon Press, Oxford, U.K.1981.
4. E.H.Shortliffe, Computer-Based Medical Consultations: MYCIN, American Elsevier Publishing Inc., 1976.
5. B.G.Buchanan & E.H.Shortliffe, Rule-Based Expert Systems, Addison-Wesley Publishing Co., Inc., Massachusetts, 1984.
6. S.XU. Current Status and Prospects of Theoretical Research of the AHP, J. of Decision & Analytic Hierarchy Process, NO.1. pp.35-41, 1989.(in Chinese)
7. N.I.Nilsson, Principles of Artificial Intelligence, Tioga Publishing Co., 1980.
8. P.H.Winston, Artificial Intelligence, 2ed. Addison-Wesley. Publishing Company,1984.
9. Zhu Song-chun, Systematic Thinking And Modem Organization

Management, Guang Ming Daily Press, Beijing, China, 1988. (in Chinese)

10. Zhu Song-chun & Sun Zhi-rong, Practical Decision-Making, P.L.A. Press, Beijing,China, 1988.(in Chinese)

11. Zhu Song-chun & Gu Ji-fa, System Engineering And Leadership, Shan Dong People's Press, Shan Dong, China, 1986.(in Chinese)

12. Zhu Song-chun (ed.), System Engineering And Modem Scientific Management, Beijing Society of Sciences, Beijing, China, 1986.(in Chinese)

13. Zhu Song-chun & Wan Xiang, Uncertainty Principle of Complex System (UPCS) and It's Applications to Economy, Selected Works (I) of International Conference on the Decision-Making Modeling of the Great (World) System the Chinese Philosophy and Global Modeling, Far Eastern Institutes of Advanced Studies, U.S.A., Aug.1988.

14. Zhu Song-chun & Jin Min, The New Development of the Application of the Combat Power Scores, Proceedings of China/United States Seminar on Methodologies in Defense System Analysis, Beijing, China 1987.

15. Wang Hong, Zhu Song-chun & Song Chun-li, Evaluating And Analyzing the Combat Capabilities of Six Countries' Mechanized Divisions, J. of Decision and Analytic Hierarchy Process, NO.2, pp. 30-41, 1990.