THE APPLICATION OF HIERARCHICAL STRUCTURE WITH INNER DEPENDENCE TO THE EVALUATION OF KEY PRODUCTS MADE IN TIANJIN

Guo Fan Xu SHu-bo Gao Zi-guang Chu Lin

Department of Engineering Economics & System Engineering, The Research Insitute of System Engineering, Tianjin University, China

ABSTRACT

Some decision-making problems need to be free from assumptions that all elements within a level are inner independence. This paper deals with how to calculate the priorities of hierarchical structures with inner dependence, and by using it, we can solve the indexes weighting of the evaluation model of the key products of Tianjin.

I. Introduction

The Raise of the Question

In order to implement the strategy of the coastal economy, from now on till the year 2000, Tianjin's economy will be faced with a very important turn of which the rational adjustment of the industrial structure is an impontant part. How is the adjustment done? It is crucial to choose the right direction for the adjustment of the industrial structure. However, the determination of the direction must be based on the actual situation, overall evaluation of the advantages and disadvantages, feasibility and needs etc. This paper tries to evaluate the potentialities of 60 or so key products of Tianjin in order to provide a scientific and objective basis for determing the right direction of the economic development.

The Study of the Methods

It is a multi-criterion problem of decision making to give the priority to the comprehensive superiority of the development of various key products of Tianjin which should be generally evaluated by means of the economic indexes. The diffenent degrees of influence by the different indexes to the development take it impossible to make the directly quantitative measurement of the relative importance. The analysis hierarchy process however can effectively combine the qualitative analysis with quantitative analysis. i.e. by means of people's judgement, the various alternatives can be given priority according to their advantages and disadvantages, which is prictical multi-criterion method.

The hierarchical structure with inner independente is the simplest structure system. With the complication of decision-making problems, different kinds of structures will come into being. This paper discuss some patterns of the impact priority of different structure. and deduce particularly calculating methods of the comosition priority to the

hierarchical structure with inne dependence.

In terms of the characteristics of the practical evaluation model of the key products of Tianjin, the hierarchical structure with inner dependence has been established to determine the indexes weighting, and therefore. to solve the problem of evaluation.

II. Hierarchical Structure wiht Inner Dependent

In the course of solving the problems in decision-making, the hierarchical structure wijth inner independent can be used to simplify the system when the interactions between the elements within the same level are so weak as to be neglected. However, for some complicated problems, the interactions between the elements within the same level should not be neglected. Otherwise, the result will influence the degree of the real fact. In order to establish a good model of systems. the correlative influence between the elements must be take into consideration so that the hierarchical structure with inner dependence can be established.

Some Patterns of Impact Priority

The influence of an element to another is called impact priorities. The influence of the hierarchical structure with Inner dependence appears between the elements in a lower level and those in any higher level, and moreover an infllence among elements within a same level appears, which forms feedback loops through which the interaction may appear once up to infinite. The limiting impact priority is needed.

In the the hierarchical structure with inner independence the elements of a higher level are not influential to those of any lower level which, in reverse, are subzect to those of any higher level. Moreover, the impact function has only one path, and finite number of times, i.e. accumulated impact function is finite, which can be proved easily by supermatrix: The supermatrix of n levels is as follow:

$$W = \begin{bmatrix} 0 & & & & \\ w_{21} & 0 & & & \\ & w_{32} & 0 & & \\ & & \ddots & \ddots & \\ & & & w_{nn-1} & I \end{bmatrix} \qquad W^{k} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ w_{nn-1}\cdots w_{21} & w_{nn-1}\cdots w_{32} & \cdots & I \end{bmatrix}$$

$K>n-1$ $W^{k}=W^{k+1}$ It proves that the impact function only has finite n-1 steps, but does not have other more than n-1 steps when there have n level. If the number of levels is determined, the accumulated impact function fix. The influence of n-1 steps can only appear in the lowest level with respect to the highest. The result of which will be exactly the same as the one of the composition of priority, indicating the real priority of the alternatives.

In other structures such as cycle structure and the general feedback system, there exist feedback loops. amony levels or amony elements within a level. The level are not labeled hiyher or lower, because they both

directly or indirectly dominate others or be dominated by others. The principle of the composition of priority is no longer suitable, and the limiting impact priorities should be considered.

In different structures there are different patterns of impact priority Generally, if the system has a top level, the composition of priority can be obtained on the basis of the limiting impact priority. So hoerarchical seructure with inner dependence can have a composition of priority on the basis of the limiting impact priority.

The Composition of Priority of the Hierarchical Structuture with Inner Dependent.

1. The interactions occur only among the elements on the alternatives level.

Supermatrix is as follow.

$$W = \begin{bmatrix} 0 & & & & \\ W_{21} & 0 & & & \\ & W_{32} & 0 & \cdots & \\ & & \ddots & \ddots & \\ & & & W_{nn-1} & W_{nn} \end{bmatrix}$$

The limiting impact priority:

$$\lim_{k\to\infty} W^k = W^\infty = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ W^\infty_{nn}W_{nn-1}\cdots W_{21} & W^\infty_{nn}\cdots W_{32} & & W^\infty_{nn}W_{nn-1} & W^\infty_{nn} \end{bmatrix}$$

When the limiting impact priority W_{nn} can be gotten, the limting impact priority W is obtained.

[1] When W_{nn} is a primitive stochastic matrix.there is:

$$W^\infty_{nn} = \frac{C(1)}{R'(1)}$$

When $R(1)$ is W_{nn}'s minimal polynomial, and $C(r)=(rI-W_{nn})^{-1}R(r)$ is the reduced adjoint matrix.

[2] When W_{nn} can be given by:

$$W_{nn} = \begin{bmatrix} Q_1 & & & 0 \\ & \ddots & & \vdots \\ & & Q_m & 0 \\ U & \cdots & & q \end{bmatrix} \qquad W^k_{nn} = \begin{bmatrix} Q_1^k & & & 0 \\ & \ddots & & \vdots \\ & & Q_m^k & 0 \\ U_k & & & q^k \end{bmatrix}$$

Where Q_i $(i=1\cdots\cdots m)$ is a primitive stochastic matrix, and the moduli of the characteristic roots of Q care strictly less than 1 :

$$Q_i^{\infty} = \lim_k Q_i^k = \frac{C_i(1)}{R_i'(1)} \qquad (i=1\cdots\cdots m)$$

$$q^{\infty} = \lim_k q^k = \bar{0}$$

$$U_{\infty} = Q_i U_i (I-q)^{-1}$$

$$W_{nn}^{\infty} = \begin{bmatrix} Q_1^{\infty} & & & 0 \\ & \ddots & & \vdots \\ & & Q_m^{\infty} & 0 \\ & U_{\infty} & & 0 \end{bmatrix}$$

[3] When number c is the period of given W_{nn}, there is as follow:

$$W_{nn}^{\infty} = \frac{1}{c}\left(I + W_{nn} + W_{nn}^2 + \cdots\cdots W_{nn}^{c-1}\right)\left(W_{nn}^{c}\right)^{\infty}$$

[4] When W_{nn} can be given by:

$$W_{nn} = \begin{bmatrix} A_1 & & & 0 \\ & \ddots & & \vdots \\ & & A_m & 0 \\ & U & & P \end{bmatrix} \qquad W_{nn}^k = \begin{bmatrix} A_1^k & & & 0 \\ & \ddots & & \vdots \\ & & A_m^k & 0 \\ & U_k & & P^k \end{bmatrix}$$

where $A_i(i=1.....m)$ is cycle matrix with number c as a period, and the moduli of the characteristic roots of P are strictly less than 1, thus

$$A_i^{\infty} = \frac{1}{c}\left(I + A_i + A_i^2 + \cdots\cdots + A_i^{c-1}\right)\left(A_i^{c}\right)^{\infty} \qquad (i=1\cdots\cdots m)$$

$$P^{\infty} = \lim_{k\to\infty} P^k = 0$$

$$W_{nn}^{\infty} = \lim_{k\to\infty} W_{nn}^k = \begin{bmatrix} A_1^{\infty} & & & 0 \\ & \ddots & & \vdots \\ & & A_m^{\infty} & 0 \\ & U_{\infty} & & 0 \end{bmatrix}$$

2. The interactions occur only among the elements within criteria levels

Supermatrix is as follow:

$$W = \begin{bmatrix} a_{11}w_{11} & & & & \\ a_{21}w_{21} & a_{22}w_{22} & & & \\ & a_{32}w_{32} & \ddots & & \\ & & \ddots & \ddots & \\ & & & a_{nn-1}w_{nn-1} & I \end{bmatrix}$$

It can be proved by induction:

$$W^{\infty} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ d_1 & d_2 & \cdots & d_{n-1} & d_n \end{bmatrix}$$

$$d_1 = a_{nn-1}w_{nn-1}\,(1-a_{n-1n-1}w_{n-1n-1})^{-1}\cdots\cdots a_{21}w_{21}(1-a_{11}w_{11})^{-1}$$

$$d_2 = a_{nn-1}w_{nn-1}\,(1-a_{n-1n-1}w_{n-1n-1})^{-1}\cdots\cdots a_{32}w_{32}(1-a_{22}w_{22})^{-1}$$

$$\vdots$$

$$d_{n-1} = a_{nn-1}w_{nn-1}\,(1-a_{n-1n-1}w_{n-1n-1})^{-1}$$

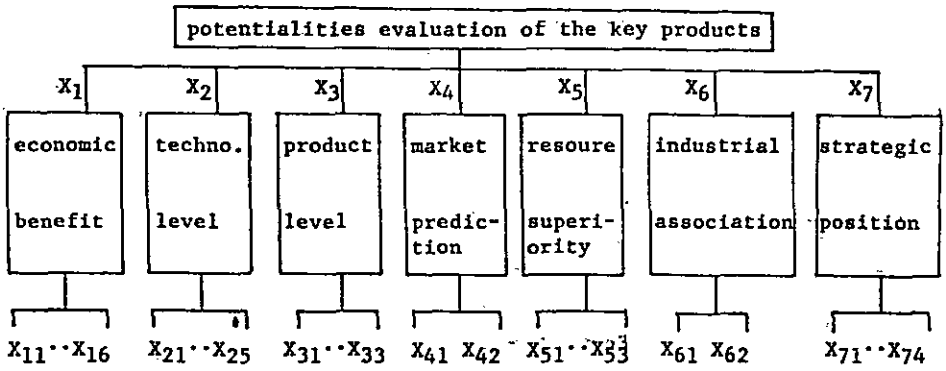
$$d_n = I$$

d_1 is the composition of priority

3. The interactions occur among the elements criteria' levels and alternatives level

In this case, the priority's calculating formula can be done which is the same as that of gerenal feedback system.

III. Practical Model Structure



X_1. Economic benefit
(X_{11}) labour productivety
(X_{12}) benefit/output ratio
(X_{13}) output/funds ratio
(X_{14}) benefit/funds ratio
(X_{15}) consumption/output ratio
(X_{16}) equibibrium capability of exchange

X_2. Technological level
(X_{21}) New degree of production facility
(X_{22}) Level of prodaction engineering and production facility
(X_{23}) The rate of technical progress contribution to the rate of output increase
(X_{24}) Capability of technililgical development
(X_{25}) The proportion of technical personnel.

X_3. Product level
(X_{31}) The porportion of occupancy market
(X_{32}) The porportionof high-class products
(X_{33}) The porportion of export products

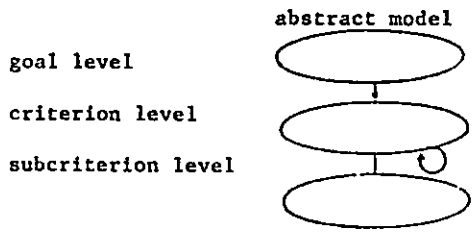
X_4. Market prediction
(X_{41}) The index of supply and demand in world market
(X_{42}) The index of supply and demand in domestic market

X_5. Resource superiority
(X_{51}) The proportion of supplied by Tianjin's own
(X_{52}) The proportion of supplied by domestic market
(X_{53}) The proportion of supplied by world market

X_6. Industrial association
(X_{61}) incentive coefficient
(X_{62}) influential cofficient

X_7. Strategic position
(X_{71}) Country's planning
(X_{72}) Economic Zone planning
(X_{73}) Tianjin's planning
(X_{74}) Industry planning

In the view of systematicness, the over 60 key products of Tianjin are evaluated from the following seven aspects: economic benefit, technological level, product level. resource superiority, market prediction, industrial association strategic position. Obviously, these seven aspects are not independent, but innerdependent on each other. This is to say that the interaction within criterion level forms the hierarchical structure with inner dependence.



The supermartrix is as follow:

$$W = \begin{bmatrix} 0 & 0 & 0 \\ w_{21} & a_{22}w_{22} & 0 \\ 0 & a_{32}w_{32} & I \end{bmatrix} \quad W^{\infty} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_{32}w_{32}(1-a_{22}w_{22})^{-1}w_{21} & a_{32}w_{32}(1-a_{22}w_{22})^{-1} & I \end{bmatrix}$$

The composition of priority is $d_1 = a_{32}w_{32}(1-a_{22}w_{22})^{-1}w_{21}$

Take mechanical and electronical products for example. By using the above formula deduced the indexes weighting can be arranged. All judgement matrix are be given by four experts.

Step 1. The pairwise comparison matrix of these seven aspects is estabilished according to their impact on the focus. The priority vector is derived from the juydgement matrix

$$V=(0.24973,\ 0.18689,\ 0.12677,\ 0.19878,\ 0.08133,\ 0.10834,\ 0.04809)^T$$

Step 2. The interdependence within the criterion level is analized. The pairwise comparison matrices of elements are be established to their impact in the same level. They are respectively as follows

$$V_1=(0.38645,\ 0.13953,\ 0.15544,\ 0.06252,\ 0.08537,\ 0.08991,\ 0.08075)^T$$

$$V_2=(\ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ )^T$$

$$V_3=(\ 0\ \ 0.24207,\ 0.67154,\ 0\ \ 0\ \ 0.08639,\ 0\ )^T$$

$$V_4=(\ 0\ \ 0\ \ 0.16840,\ 0.63591,\ 0\ \ 0.19569,\ 0\ )^T$$

$$V_5=(\ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ )^T$$

$$V_6=(\ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ )^T$$

$$V_7=(\ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ )^T$$

Step 3. The priorities of the elements within subcriterion level with respect those within criterion level.

$$V_{11}=(0.07157,\ 0.20121,\ 0.14269,\ 0.34508,\ 0.07406,\ 0.01654)^T$$

$$V_{22}=(0.12083,\ 0.41927,\ 0.22399,\ 0.15066,\ 0.08526)^T$$

$$V_{33}=(0.44771,\ 0.34356,\ 0.20874)^T$$

$$V_{44}=(0.33333,\ 0.66667)^T$$

$$V_{55}=(0.44771,\ 0.34356,\ 0.20874)^T$$

$$V_{66}=(0.75000,\ 0.25000)^T$$

$$V_{77}=(0.47102,\ 0.23678,\ 0.19932,\ 0.09929)^T$$

Step 4. Weighting matrix of which the subcriterion level is compared with criterion level

$$A=(a_{22},\ a_{32})=(0.25,\ 0.75)$$

The composition of priority in this model is as follow:

$$W_{21} = V_{7X1}$$

$$W_{22} = (V_1 \quad V_2 \quad V_3 \quad V_4 \quad V_5 \quad V_6 \quad V_7)_{7X7}$$

$$W_{32} = \begin{bmatrix} V_{11} & & & & & & \\ & V_{22} & & & & & \\ & & V_{33} & & & & \\ & & & V_{44} & & & \\ & & & & V_{55} & & \\ & & & & & V_{66} & \\ & & & & & & V_{77} \end{bmatrix}_{25X7}$$

$$W = a_{32}w_{32}(1-a_{22}w_{22})^{-1}w_{21}$$

$$=(0.01484,\ 0.04172,\ 0.02958,\ 0.07154,\ 0.01535,\ 0.03429,\ 0.02505,\ 0.08691,\ 0.06443,\ 0.03123,\ 0.01767,\ 0.04825,\ 0.02596,\ 0.05889,\ 0.06037,\ 0.12074,\ 0.03906,\ 0.02997,\ 0.01821,\ 0.09765,\ 0.03255,\ 0.02528,\ 0.01271,\ 0.01699,\ 0.05329)^T$$

From above, The indexes weighting of the evaluation model of mechanical and electrical products are obtained, by which the practical indexes can be weighted. The total scores may be obtained, which can be used to evaluate the prospect of the products.

IV. Conclusion

In this paper, some kinds of patterns of the impact priority of different structures are discussed. Except for the hierarchical structure withe inner independence, limiting impact priority must be considered in other structures. The calculating formula of the complsition of priority about hierarchical structure with inner dependence are deduced. By using the formula deduced to determine the indexes weighting. The evaluation provblem of key products can be solved.

Reference

[1] F.R. Gantmacher "Applications of the Theory of Matrices" Interscience Publishers, INC. NEW YORK, 1959

[2] Liu Lin "The priority methods of feedback system" 1987

[3] Tan Tian-shun "Technical progress measurement of economic system" 1986

[4] T.L. Satty "Dependence and Inedpendence: From linear Hierarchies to Nonlinear Networks" 1986

[5] T.L. Satty "The Analytic Hierarchy Process" 1980

[6] T.L. Satty "The General Case of Dependence in Hierarchic Decision Theory" 1986

[7] Xu Shu-bo "The principle of Analytic Hierarchy Process"