ISAHP 1996, Vancouver, Canada, July 12-15, 1996

AN APPLICATION OF AHP METHOD TO MARKOV DECISION PROCESS

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INTRODUCTION

It is very important to carry out macroscopic control for industry economy in the course of the operation of the market mechanism. This paper introduces the integrated model of AHP method and Markov decision process so as to provide the accordance and information on quantitative analysis to make an economic development plan.

1. PROBLEMS

In case, a decision department has some subordinate enterprise groups, it puts respectively two kinds of management policies to the subordinate enterprises. The first, it transfers its power to the enterprises that accept the market regulation; the other, it implements the instructive policy of macroscopic control and manages all the things done by its subordinates. These two kinds of management policies are assumed as U_1 and U_2 that represent respectively the two parts of ratio relationship. If $U_1+U_2=1$, $U_1, U_2 \geqslant 0$, the problem is what ratio U_1 and U_2 should respectively possess in the total industry.

2. ESTABLISHMENT OF MATHEMATICAL MODEL

There are more than three thousand subordinate enterprises in Shenyang city, Liaoning Province, China. If all of them operate entirely according to the market regulation, many enterprises will have not any adaptive ability and go into bankruptcy; if all of them operate entirely according to the planned economy, the government in Shenyang has no capability enough. For this reason we apply the above-mentioned two kinds of policies U_1, U_2. We have analyzed the state of these enterprises, and we divide them into five circumstances according to our investigation:

S_1: developing rapidly; S_2: developing proportionally; S_3: remaining the status quo; S_4: shrinking 30%; S_5: shrinking more than 70%.

We establish the network structure model as follows:

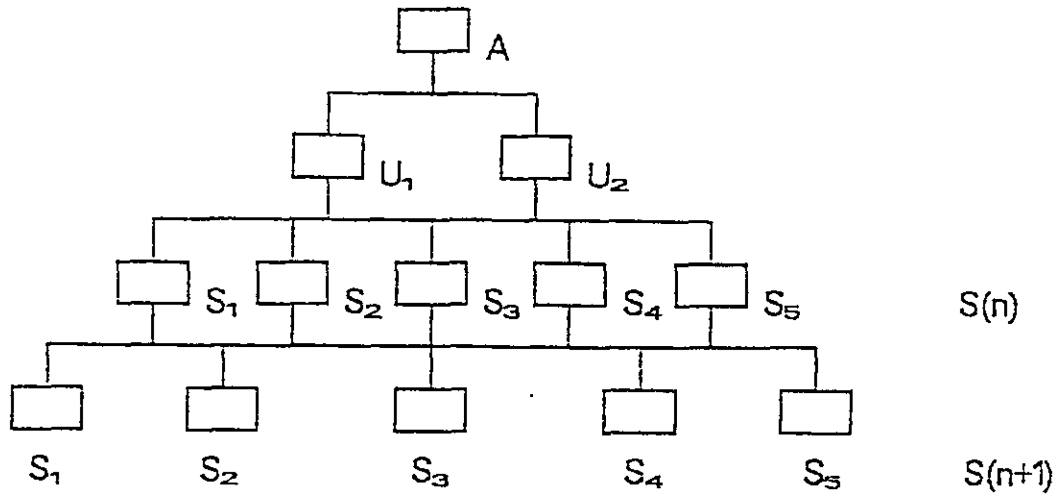

Figure 1 Network Structure

Assuming that $U=(U_1, U_2)^T$, then. $S(n)=\{S_1(n), S_2(n), \ldots, S_n(n+1)\}^T$, and $S(n+1)=\{S_1(n+1), S_2(n+1), \ldots, S_n(n+1)\}^T$.

In this formula, $S=(S_1, \ldots, S_m)$ is called a state vector, n is time which is measured by the year, $m=5$, $S_1, \ldots, S_5$ indicates five sorts of states mentioned above.

The transfer probability matrix from state S(n) to S(n+1) is
$\{P_{ij}(n)\}$, $i=1,2,\ldots,5$; $j=1,2,\ldots,5$.

The probability distribution of state S(n) in U_1 is $\beta_{11}(n)$, $\beta_{12}(n)\ldots$, $\beta_{15}(n)$.

The probability distribution of state S(n) in U_2 is $\beta_{21}(n)$, $\beta_{22}(n)\ldots$, $\beta_{25}(n)$.

U_1 and U_2 are the policies adopted by the decision department A. Through Markov process, it is known:

$W(n+1)=P(n)\,\beta(n)(U_1, U_2)^T$. In this formula. W(n+1) is the probability distribution of S(n+1).

It is assumed that the profit vector of $S(n)=[S_1(n), S_2(n), \ldots, S_5(n)]$ is $Q(n)=[Q_1(n), Q_2(n), \ldots, Q_5(n)]$; then, the total profit each year is

$$C(n)=\sum_{i=1}^{5} W_i(n)Q_i(n)$$

3. THE ESTIMATED PARAMETER FOR AHP ---- β(N), P(N)

In object U_1, the relative weighted value of $S_1, \ldots, S_5$ is $\beta_{11}(n), \ldots,$

$\beta_{15}(n)$; in object U_2, the relative weighted value of S_1, S_2, ..., S_5 is $\beta_{21}(n)$, ... $\beta_{25}(n)$. Similarly, the relative weighted value of $S(n+1)$ in $S(n)$ can be estimated with AHP method, and the matrix $\{P_{ij}(n)\}$, $i,j=1,2,....,m$, is obtained.

It is assumed that $U_2=X$, as a result, $U_1=1-X$, here, $0 \leqslant X \leqslant 1$; then, $W(n+1)=P(n)\,\beta(n)\,(U_1, U_2)^T$ can also be known in U+1 years. The total profit of each year is

$$C(n)=\sum_{i=1}^{5} W_i(n)Q_i(n)$$

The parameter in the Markov decision process can completely be estimated with AHP method, therefore, the analysis for Markov decision process can be carried out.

4. DECISION ANALYSIS ALGORITHM

Markov decision process is shown in the following figure:

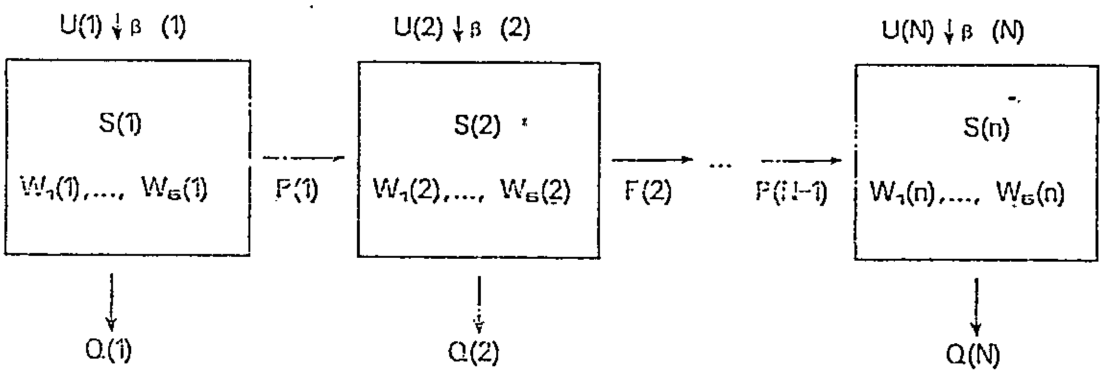

Figure 2 Markov Decision Process

In Figure 2, n=1 is an initial state, which indicates one year; when n is equal to 2, the relative weighted value of S(2) in U(2) is $\beta(2)$. Through the transfer matrix of P(1) from S(1) to S(2), $W(2)=P(1)\,\beta(1)\,(U_1, U_2)^T$ can be calculated. After that, base on the calculated $\beta(2)$, P(2) and the given U_1, U_2, W(3) can also be calculated. Similarly, the expression W(4), W(5) can be calculated in turn. Because $U_2(n)=X_n$, $U_1(n)=1-X_n$ is assumed each time, the state equation for making an n year plan is :

$W(n+1)=P(n)\,\beta(n)\,(U_1(n), U_2(n))^T$;

Object function:

$$J_N=\sum_{n=1}^{5} C(n), \quad C(n)=\sum_{i=1}^{5} W_i(n)Q_i(n)$$

According to Bellman dynamic programming,

$$J^*_{n-j} = \underset{0<X<1}{Max}\{C_{n-j} + J^*_{n-j+1}\}, \quad j=0,1,2,\ldots,n$$

So, $U_2(n)=X_n$, $U_1(n)=1-X_n$ can be solved as the basis for decision department to make a policy.

5. CONCLUSION

In accordance with the investigation and calculation in three thousand enterprises in Shenyang, Liaoning Province, China,

$W(n)$, $Q(n)$, $n=1,2,3,4,5$, is obtained.

The optimal solution of $\{U_2(n)=X_n\}$ $\{U_1(n)=1-X_n\}$ have been obtained with Bellman algorithm in following figure :

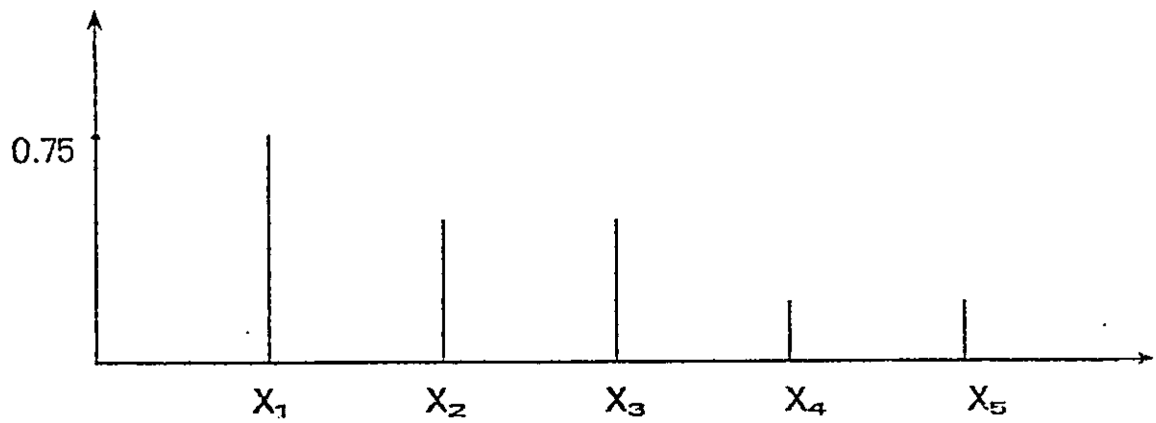

Figure 3

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