The AHP of Group Multiple Round feedback Judgement

Huiwen Diao, Weimin Zhou
Taiyuan Institute of Machinery
Shanxi, China

Abstract

In this paper, we present a method that combines the AHP and the Delphi Method. Priorities for alternatives are obtained from group judgments. In the Delphi method a quartile is used to synthesize group opinions. In addition, a new method to compute the priorities and revising judgments is proposed. The judgments are revised using a difference function. This function could be used to compare and evaluate the judgment revised. Finally, we provide an example to illustrate the approach.

1. Introduction

The Analytic Hierarchy Process is well-known in the World as a simple and practical tool for multiple criteria decision making. With new developments on decision science, the AHP will be further developed and improved.

(1) The Method

The method illustrated here is based on actual decision problems. It is best described as a group of people (usually experts), who are invited to discuss a set of alternatives for several rounds to obtain their opinions or judgements, so that a final decision can be obtained.

This situation has several characteristics in common with other disciplines:

a. Group decision making: Organizationl behavior does systematic research on group decision problems ([1], [2])

b. The nature of the selection process: People's identification of alternatives does not usually require as much time as the selection of the most desirable alternative,

c. Multiple round feedback. The actual decision process requires several rounds to obtain opinions. The feedback of the information in every round makes opinions converge gradually.

Why should one use a way of making decisions that require a group consensus?

a. Many decision problems are important, large scale, complex or strategic and require a group of people to provide alternatives and pool the wisdom of the group to ensure the correctness and quality of the decision.

b. Every round of information feedback will: (i) make the background and detail of the decision problem more distinct, (ii) make different opinions and conflict of interests gradually come to a compromise and consensus, (iii) increase agreement on the final decision and (iv) make more people feel confident and duty bound. Therefore, group consensus helps to execute decision alternatives as quickly and as efficient as possible.

d. Multiple round feedback has an error correcting mechanism. It provides a chance for correcting errors in judgments made in the preceding round. Furthermore, people often can learn many concrete decision technology and methods in the decision process.

(2) Delphi Method and AHP

The Delphi Method undoubtedly reflects the features of group decision making with multiple feedback and it has also been widespread. According to our experience, direct weight assessment methods are usually very inaccurate for evaluating a set of alternatives, the ranges of weights in every round is large, and the sequence of preferences is not is not steady.

It is necessary to distinguish between AHP with feedback and the dynamic AHP. AHP with feedback is suitable for a non-hierarchy network system, which is inner dependent on the interaction of elements within the system. In the dynamic AHP, the judgement matrix and its weights are all continuous functions of time (i.e.

A(t), W(t)). The concepts "dynamic or time dependent" used in the dynamic AHP has the meaning of "continuous" versus "discrete".

AHP has paid attention to this decision method. Saaty [3] (pp.60-70) points out that experts operate in a group discussion face to face and that a judgment matrix is only obtained after opinions relatively sharpen. Dalkey has pointed out that the back to back way is better than the face to face way in group decision making. Researching organizational behavior also indicates that with spoken language, expression and movement, face to face communication will create the obstacles of communication, affect information expression, conveyance and understanding, and finally affect quality of decisions. When the ability of expression is not strong, the enunciation is not clear, the meaning is fuzzy or the information is disturbed, one's confidence may be lost. Therefore, there is a new task to study the way group decisions are conducted in AHP (such as change the face to face discussion to the back to back discussion, and express the information feedback in a mathematical language).

Although Saaty [3] (pp. 66-70) has discussed some differences between AHP and the Delphi Method based on the above analysis, it is possible to combine these two methods to address the problems mentioned above.

2. A Method to Synthesize Group Opinion and Obtain Priorities

(1) A method to synthesize group opinion

In this paper, we use in the Delphi Method a quartile for synthesizing group opinions. The calculation of quartiles is as follows:

- Arrange the paired comparison judgments $a^k_{ij}(k=1,2,\ldots,n)$ in increasing order.
- Let the lower quartile be M_{ij1}, the median M_{ij0}, and the upper quartile be M_{ij2}.

Let

$n = 2p+1$ if n is odd

$n = 2p$ if n is even

then

$$M_{ij1} = \begin{cases} a_{ij}^{(p+1)/2} & \text{if } p \text{ is odd} \\ (1/2)\, a_{ij}^{p/2} + a_{ij}^{(p/2)+1} & \text{if } p \text{ is even} \end{cases}$$

and

$$M_{ij1} = \begin{cases} a_{ij}^{p+1} & \text{if } n \text{ is odd} \\ (1/2)\,[a_{ij}^{p} + a_{ij}^{p+1}] & \text{if } n \text{ is even} \end{cases}$$

and

$$M_{ij2} = \begin{cases} a_{ij}^{(3p+3)/2}, & \text{if } n,\ p \text{ odd} \\ (1/2)\,[a_{ij}^{3p/2+1} + a_{ij}^{3p/2+2}], & \text{if } n \text{ odd, } p \text{ even} \\ a_{ij}^{(3p+1)/2}, & \text{if } n \text{ even, } p \text{ odd} \\ (1/2)\,[a_{ij}^{3p/2} + a_{ij}^{3p/2+1}], & \text{if } n,\ p \text{ are even} \end{cases}$$

The parameter triad (M_{ij1}, M_{ij0}, M_{ij2}) reflects the statistical distribution of the information of expert opinions.

Some methods to synthesize group judgement include the arithmetic mean, the geometric mean, the harmonic mean, the root-square-mean, the root-power-mean, the exponential mean, the statistical method based on lognormal or normal distribution, R-fuzzy set method, etc... [5], [6], [7]. The first two methods are used often. Although the above methods are perfect in theoretical grounds, the cost and energy taken in a complex algorithm and the large amount of calculations necessary will offset these advantages. Moreover, these methods usually require computer support.

In Table 1, the geometric mean (GM), arithmetic mean (AM) and quartile method are compared. According to this table we obtained following results:

a. The standard deviation σ_{ij} of the three methods are roughly the same. If the criterion to synthesize judgments is the Euclidean distance then these methods have approximately the same precision.

Table 1

Three Methods of Synthesizing Group Judgments

section number	group judgement a_{ij}^{k}	GM		AM		quartile	
		$\bar{a}_{ij}$	σ_{ij}	$\bar{a}_{ij}$	σ_{ij}	M_{ij0}	σ_{ij}
I	1,2,3,3,3,4,4,5,5	3.333	1.323	3.031	1.361	3	1.369
II	$\frac{1}{7},\frac{1}{7},\frac{1}{5},\frac{1}{5},\frac{1}{5},\frac{1}{5},\frac{1}{4},\frac{1}{3}$	0.209	0.006	0.202	0.006	0.2	0.0066
III	1,2,3,3,3,4,4,4,5,7	3.6	1.647	3.223	1.694	3.5	1.65
IV	$\frac{1}{5},\frac{1}{3},\frac{1}{3},\frac{1}{3},\frac{1}{3},\frac{1}{2},\frac{1}{2},9$	1.442	3.056	0.523	3.21	0.333	3.277
V	1,1,1,1,1,1,1,2,7	1.778	1.986	1.341	2.039	1	2.151
VI	1,7,9,9,9,9,9,9,9	8	2.539	7.045	2.731	9	2.749

b. The quartile method gives the user the ability to automatically reject extreme judgements, such as the "7" in section V and the "1" in section VI. Through the senses, medians equal to "1" in section V and "9" in section VI as the synthesized judgments are more reasonable.

c. The synthesis in GM and AM is affected by extreme judgments, particularly in section V. According to direct perception, the means of GM and AM all deviate from the main part of group judgment. To reject extreme judgments in the two methods, the steps and processes would be very complex [8].

d. If group judgments are normally (or lognormal) distributed, such as in section I, II, and III, then the results are close.

e. The three statistical parameters of the quartiles provide more useful information than those of GM and AM. For example, in section II, the triad information (0.171, 0.2, 0.225) can directly reflect the group judgment distribution, the interval [0.171, 0.225] contains 50% of the experts opinions. let

$$\Delta M_{ij1} = M_{ij0} - M_{ij1} = 0.029$$
$$\Delta M_{ij2} = M_{ij2} - M_{ij0} = 0.025$$

ΔM_{ij1}, ΔM_{ij2} directly reflect the degree of expert opinion in both sides of the median.

However, σ_{ij} in the other methods can only abstractly reflect the shape of the distribution. Thus, the information from the quartiles can efficiently help experts to revise judgments.

Aczel and Alsina ([9], [10]) have proposed conditions for synthesizing group judgement which involve separability, unanimity and reciprocal property. In [11] we have proven that the quartiles satisfy these conditions.

(2) Computation of the priority

Song Yuanfang [12] has attempted to combine AHP with Delphi Method, but his algorithm has the following problems:

a. The number of feedback rounds is not restricted. It is possible to go through three or four rounds as in the Delphi Method, thus decreasing the quality of judgements because of the experts may get tired.

b. Every feedback information parameter is part of the matrix (i.e. $\bar{a}_{ij}$, σ_{ij}, when $\sigma_{ij} > \epsilon(n)$). Although repetitive work in this process is small, the revision of some of the elements could endanger the entire set of judgements.

Due to these problems, we consider that the number of feedbacks must be restricted, and every bit of information in the matrix must be feedback. We suggest the following steps to compute the priorities:

step 1: Select the experts.

step 2: Discuss and construct a hierarchic model for the decision problem by the decision makers, analysts or experts.

step 3: Send blank comparison matrix or questionnaire to experts, and receive n pieces of the comparison matrix. The matrix data are: $\{ a^k_{ij} \}$, i, j = 1,2,...,m. k = 1,2,...,n

Where a^k_{ij} is the pairwise comparison of alternative i versus alternative j provided by the kth expert.

(Synthesizing group judgments using quartiles.)

step 4: Send the triad (M_{ij1}, M_{ij0}, M_{ij2}) feedback to the experts. Let them correct their judgement according to the statistic information and the above round judgement, and record the new round judgement data.

step 5: Repeat step 3, step 4 for less than 4 rounds, using the median M_{ij0} of the last round as the element a_{ij} of the final judgement matrix in the entire choice process, i.e. $a_{ij} = M_{ij0}$, $A = \{ a_{ij} \}$.

step 6: Compute the priority and consistency for the matrix A, and obtain the eigenvector W, λ_{max}, CR, and son on.

3. Revising Judgments and Evaluating the Total Difference

There are a number of methods for revising an inconsistent judgement matrix ([13],[14].) Saaty proposed an important criterion for revising judgements:"One would rather have naturally improved judgements arising from experience". According to this criterion, the method which is truly reasonable for revising judgement still needs research.

(1) An new method for revising the judgment matrix.

Saaty [3] (PP65-66) suggested three methods warranting judgment revision:

a. Form the matrix of absolute differences $\{ | a_{ij} - w_i/w_j | \}$ judgments on the element(s) or row sums with the largest such differences.

b. Get the root mean square deviation using the rows of $\{a_{ij}\}$ and $\{ w_i/w_j \}$, revise the judgement for the row with the largest value.

c. Revise judgements related to selecting the largest of the ratio of a_{ij} to w_i/w_j.

These methods can be repeated until consistency reaches an acceptable level. Saaty also pointed out: "Excessive use of this method of forcing the value of a judgment to improve consistency distorts the answer".

Based on Saaty's criteria and methods, we propose a new method of revising the group decision approach. The steps are as follows:

step 1: Form the matrix of differences $\{ \Delta a_{ij} \}$
where $\Delta a_{ij} = a_{ij} - w_i/w_j$
let $| \Delta a_{1t} | = \max_{i,j} | \Delta a_{ij} |$

Thus, the element with the largest absolute difference is in the entry (1,t).

step 2: Let $\Delta a_{1t} > 0$,
if $w_i/w_j \in [M_{1t1}, M_{1t0}]$, then $a'_{1t} = w_i/w_j$,
if $w_i/w_j < M_{1t}$, then $a'_{1t} = M_{1t1}$.

step 3: Let $\Delta a_{1t} < 0$,
if $w_i/w_j \in [M_{1t0}, M_{1t2}]$, then $a'_{1t} = w_i/w_j$,
if $w_i/w_j > M_{1t2}$, then $a'_{1t} = M_{1t2}$.

step 4: If $a_{1t} \in$ set $\{ M_{1t1}, M_{1t2} \}$ and $w_i/w_j \notin [M_{1t1}, M_{1t2}]$, then do not alter a_{1t} and find a replacement for the element that has the second largest absolute difference.

step 5: Priority and consistency text.
The above steps can be repeated until consistency is met.
The other two methods proposed by Saaty can be modified accordingly.

The main idea of the new method is that the revision must be done in the quartile interval in accordance with the real meaning of the decision problem. The reasons are as follows:

a. $a'_{ij} \in \{ 1/9, 1/8, \ldots, 8, 9 \}$ can satisfy AHP ratio scale. If $w_i/w_j > 9$ or $w_i/w_j < 1/9$, and a_{ij} is replaced by w_i/w_j, although it may not affect the calculation, it can not be directly explained, and moreover, it is contrary to the ratio scale in AHP.

b. $a'_{ij} \in [M_{ij1}, M_{ij2}]$ will not deviate far from the center of group opinions nor misrepresent the group opinion. Thus, the revised judgement only slightly deviates from expert group opinions.

The above analysis is another reason for us to insist on the use of quartiles.

(2) Total difference estimation and evaluation of the revision

It may be reasonable to assume that one tends to get the right judgement, and thus one does best by avoiding errors or large errors. Thus, we assume that a small deviation from the original judgment is okay to correct an erroneous judgment. However, a larger deviation may distort a normal judgment. For example, if $a_{ij} = 7$ is replaced with $w_i/w_j = 8$, then 8 will be regarded as the right judgment; if $a_{ij} = 7$ is replaced with $w_i/w_j = 12$, then we not only correct an erroneous judgment, but over correct it, and hence we add distorted information also, because the normal judgement will not be $a_{ij} > 9$.

The following total difference function before and after revision is defined to quantitatively analyzing details, which each method correct or distorts. This function can be used to evaluate and compare various methods.

Let $A = \{ a_{ij} \}$ be the original synthesis matrix, and let $w = \{ w_1, w_2, \ldots , w_n \}$ be the priority vector obtained from A. Let $s = \{ s_1, s_2, \ldots , s_m \}$ be the priority vector of the revising method 1, and let $S = \{ s_{ij} \} = \{ s_i/s_j \}$. Let $Z = \{z_1, z_2, \ldots , z_m\}$ be the priority vector of revising method 2, and let $Z = \{ z_{ij} \} = \{ z_i/z_j \}$. Let

$$\sigma_{AS}^2 = \mathrm{trace}\{(A-S)\times(A-S)^T\}$$

and

$$\sigma_{AZ}^2 = \mathrm{trace}\{(A-Z)\times(A-Z)^T\}.$$

σ_{AS}, σ_{AZ} are called the total difference function of S and Z, respectively. If $\sigma_{AS} < \sigma_{AZ}$, then method 1 is considered better than method 2.

4. An Example

The above research results have been applied to a housing allocation problem. Some of the culture backgrounds of this subject are: in today's Chinese political-economic system, the housing institution embodies social welfare, i.e., building a house is invested by the state and the houses are allocated to applicants by enterprises, and household occupants pay a small amount of rent.

The house allocating method has been translated to a priority decision (which has a quantitative basis and takes into account many factors) from a leader's personal decision. For a practical hierarchy structure of house allocation, see Figure 1:

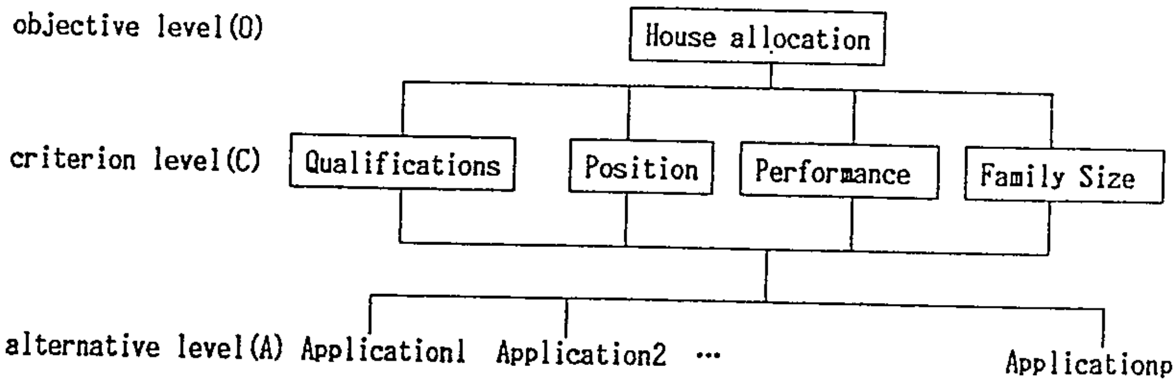

Figure 1. House allocating hierarchy

To obtain the priority weights of O-C level, we invited 12 experts to use the Delphi Method for two rounds. The quartile data of the last round is given in Table 2:

Table 2. Original judgment quartile data

O	M_{IJ1}	M_{IJO}	M_{IJ2}
c_1-c_2	1/4	1/3	1/3
c_1-c_3	1/5	1/3	5/2
c_1-c_4	4	5	5
c_2-c_3	1/5	5/2	7/2
c_2-c_4	5	5	7
c_3-c_4	5	5	7

Thus, the original synthesis matrix will be

$$C = \begin{bmatrix} 1 & 1/3 & 1/3 & 5 \\ 3 & 1 & 5/2 & 5 \\ 3 & 2/5 & 1 & 5 \\ 1/5 & 1/5 & 1/5 & 1 \end{bmatrix}$$

$$v = (0.166,\ 0.476,\ 0.301,\ 0.057)$$

$$\lambda_{max} = 4.267, \quad CR = 0.1$$

a. Using Saaty's first method, we get the following results (see Table 3). The eigenvector in the last step is given by:

$$w' = (0.189,\ 0.551,\ 0.209,\ 0.051)$$

The revised matrix is given by:

$$C' = \begin{bmatrix} 1 & 1/3 & 1/1.016 & 3.487 \\ 3 & 1 & 5/2 & 11.031 \\ 1.016 & 2/5 & 1 & 4.193 \\ 1/3.487 & 1/11.031 & 1/4.193 & 1 \end{bmatrix}$$

Table 3. The Revised Process of Saaty's Method

step	revising content	parameter(CR, $\sigma_{AC'}$)
I	w_2/w_4=8.27 replacing a_{24}=5	CR=0.061 σ=6.144
II	w_2/w_4=10.642 replacing a_{24}=8.27	CR=0.052 σ=7.49
III	w_1/w_4=3.487 replacing a_{14}=7	CR=0.033 σ=6.59
IV	w_2/w_4=11.031 replacing a_{24}=10.642	CR=0.034 σ=6.774
V	w_1/w_3=1/1.016 replacing a_{13}=1/3	CR=0.004 σ=6.626
VI	w_3/w_4=4.446 replacing a_{34}=5	CR=0.002 σ=6.38
VII	w_3/w_4=4.193 replacing a_{34}=4.446	CR=0.001 σ=6.267

b. Using our new method, we obtain the following results (see Table 4). Thee eigenvector in the last step is given by:

$$w''=(0.187,\ 0.447,\ 0.28,\ 0.056)$$

The revised matrix is given by:

$$C''=\begin{bmatrix} 1 & 1/3 & 1/1.546 & 4 \\ 3 & 1 & 1.756 & 7 \\ 1.546 & 1/1.756 & 1 & 5 \\ 1/4 & 1/7 & 1/5 & 1 \end{bmatrix}$$

Table 4. Revising process of our method

step	revising	content	parameter (CR, $\sigma_{AC''}$)	
I	$M_{242}=7$	replacing $a_{24}=5$	CR=0.071	$\sigma=5.387$
II	$M_{142}=4$	replacing $a_{14}=5$	CR=0.053	$\sigma=5.006$
III	$w_3/w_1=1.917$	replacing $a_{31}=3$	CR=0.024	$\sigma=4.917$
IV	$w_2/w_3=1.924$	replacing $a_{23}=5/2$	CR=0.014	$\sigma=4.428$
V	$w_3/w_1=1.636$	replacing $a_{31}=1.917$	CR=0.011	$\sigma=4.408$
VI	$w_2/w_3=1.756$	replacing $a_{23}=1.924$	CR=0.01	$\sigma=4.255$
VII	$w_3/w_1=1.546$	replacing $a_{31}=1.636$	CR=0.001	$\sigma=4.249$

Comparing the results of these two methods, we can see that the element a_{24} of the matrix C' goes beyond the AHP scale, and the elements a_{14} and a_{34} of C' beyond the quartile interval. Thus, these elements deviate from the center of the group opinions, and $\sigma_{AC'}=6.267 > \sigma_{AC''}= 4.249$. Therefore, the matrix C' has wide deviation, the reason being that forcing the values of the judgments distorts the answer. The matrix C" has a smaller deviation, so our proposed method seems more reasonable and sound.

The priority weights in the objective-criteria level in this example are given by:

0.187, 0.447, 0.28, 0.056

The result presented here was accepted by the decision maker. After further research, we propose that: Multiple elements with small deviations and equilibrium deviation revision of the original judgment matrix is better than that of a single element that has a larger deviation [15].

5. Conclusion

In regard to the decision method described above, there are still many new questions to be answered. In this paper we studied only a part of the entire area. Our main conclusions are as follows:

(1) The combination of AHP with Delphi Method can better solve the alternative priority problem.

(2) The quartile has some advantages: simplicity, ease of use, higher accuracy, larger information capacity, automatic rejection of extreme judgements, and realistic reasonableness.

(3) The new revising method has direct perception meaning and is more reasonable. The total difference function can be used to estimate and evaluate every revising method.

(4) The applied example indicates that many research results are practical, reasonable and efficient.

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