

Formulation of a Learning Analytical Network Process

Chen Yuh-Wen

Institute of Industrial Engineering, Da-Yeh University
112 Shan-Jeau Rd., Da-Tsuen, Chang-Hwa 51505, Taiwan
profchen@mail.dyu.edu.tw

Keywords: Analytical network process (ANP), Neural network (NN), Learning.

Summary: *The analytical network process (ANP) is proposed by Saaty (1996) to tackle the interactions among many competing criteria when ranking alternatives. The ANP consists: (a) a control hierarchy (network of criteria) and subcriteria that control the interactions; (b) The network of influences among the elements and clusters. This ANP infrastructure is designed to simulate the decision operations in a human brain; thus, the automatic learning concept inspired from neural network (NN) is applied to integrate the ANP and NN. This study explores the possibility to construct a learning ANP so that it is able to automatically moderate itself so as to reflect more dynamics in a decision environment; furthermore, some simple learning rules are deduced from a learning ANP as the time varies. These studying results are available for constructing an intelligent ANP in the near future.*

1. Introduction

The analytical network process (ANP) is originally proposed by Saaty (1996) so as to relax the strong assumption of independence among criteria in a traditional analytical hierarchy process (AHP). This ANP is formulated based on the operations of human brain; thus, it can be regarded as the interaction process within many competing criteria, which are similar to that within neurons in a human brain. In this paper, a learning ANP based on neural network (NN) concepts is explored and some simple learning rules are deduced from such a learning ANP, which is able to automatically moderate itself so as to reflect dynamics in reality. These initial results will be useful to implement an intelligent ANP in the near future.

2. Integration of ANP and NN

In this section, basic concepts of ANP and NN are introduced. Furthermore, the integration model of ANP and NN is also proposed.

2.1 ANP Concepts

The analytical network process (ANP) is originally proposed by Saaty (1996). An ANP consists: (a) a control hierarchy (network of criteria) and subcriteria that control the interactions; (b) the network of influences among the elements and clusters. Saaty introduced the concepts of ANP as follows (Saaty, 1999): assume that we have a system of N clusters or components whereby the elements in each component interact or have an impact on or are influenced by some or all of the elements of that component or of another component with respect to a property governing the interactions of the entire system. Also assume that component h , denoted by C_h , $h = 1, 2, \dots, N$ has n_h elements, which are denoted by $C_h = \{e_{h_1}, e_{h_2}, \dots, e_{h_{n_h}}\}$. The impact of a given set of elements in a component on another element in the system is represented by a ratio priority vector derived from paired comparisons in the usual way.

The following super matrix, which consists many block matrixes ($W_{i,j}$; $i, j = 1, 2, \dots, h$) is used to identify the interactive priorities of the aforementioned system (Saaty, 1999):

$$W = \begin{bmatrix} W_{11}, & W_{12}, \dots & W_{1N} \\ W_{21}, & W_{22}, \dots & W_{2N} \\ \dots & & \\ W_{N1}, & W_{N2}, \dots & W_{NN} \end{bmatrix} \quad (1)$$

and each block matrix $W_{i,j}$ is shown as follows:

$$W_{i,j} = C_j \left\{ \begin{array}{l} \left[\begin{array}{ccc} we_{i,1}, & we_{i,2}, \dots & we_{i,n_j} \\ we_{i+1,1}, & we_{i+1,2}, \dots & we_{i+1,n_j} \\ \dots & & \\ we_{n_i,1}, & we_{n_i,2}, \dots & we_{n_i,n_j} \end{array} \right] \end{array} \right. , i, j = 1, 2, \dots, h \quad (2)$$

where

C_h : the h -th criterion, which is a cluster of subcriteria;

W : the super matrix of interactive priorities for an entire system;

$W_{i,j}$: the block matrix to represent the relative importance of the i -th criterion (C_i) over the j -th criterion

(C_j); $i, j = 1, 2, \dots, h$;

we_{n_i, n_j} : the weight to represent the relative importance of the n_i -th subcriterion (e_{n_i}) over the n_j -th subcriterion (e_{n_j}); $i, j = 1, 2, \dots, h$;

Equation (1) can be regarded as the control hierarchy, various structures of interactions among criteria can be considered – please see Saaty’s work (1999).

2.2 NN Concepts

Since this paper is focused on the feasibility of the ANP integrated with NN, only the simple perceptron model is illustrated here (Minsky and Papert, 1969). The perceptron model has the following characteristics:

- A neuron has only two states: sleep and active. Use the value “0” to represent the sleep state of a neuron; on the contrary, use the value “1” to represent the active state of a neuron.
- An output of a neuron will be an input to another neuron.
- The state of a neuron (X_k) is constrained by the following linear equation:

$$\begin{aligned} X_k &= f(net_k) \\ net_k &= \sum_l rw_{kl} X_l - \theta_k \end{aligned} \quad (3)$$

where

X_k : the state of the k -th neuron, it is a binary value, i.e., 0 or 1;

rw_{kl} : the relative weight from the l -th neuron in an input-level to the k -th neuron in an output-level;

θ_k : the threshold value of the k -th neuron; if $\sum_l w_{kl} X_l \geq \theta_k$, then the k -th neuron will be stimulated to be active; otherwise the k -th neuron remained in sleep;

f : the transformation function.

The aforementioned assumption can be graphically illustrated as in Figure 1. First, the decision maker is asked to input a vector to the input-level, which has k elements (nodes). Second, these inputs will be operated among interactions among the upper level (input-level) and the lower level (output-level) through Equation (3). Finally, the computed vector can be obtained from the output-level.

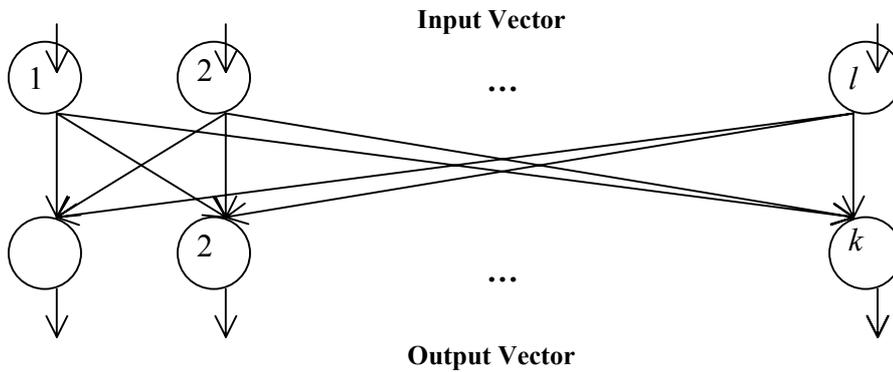


Figure 1 Operation Process of a Perceptron Model

The perceptron model has the basic ability of learning, this means it can automatically modify its relative weights (rw_{kl}) and lead to a more correct output result. This learning ability is achieved by minimizing the variance between the *computed outputs* and the *real outputs* by iterative training. The *computed outputs* are collected from numerous different input vectors, and the *real outputs* are observed by a practical survey.

2.3 Concepts of a Learning ANP

The learning ANP model is formulated by the following assumptions of (a) and (b):

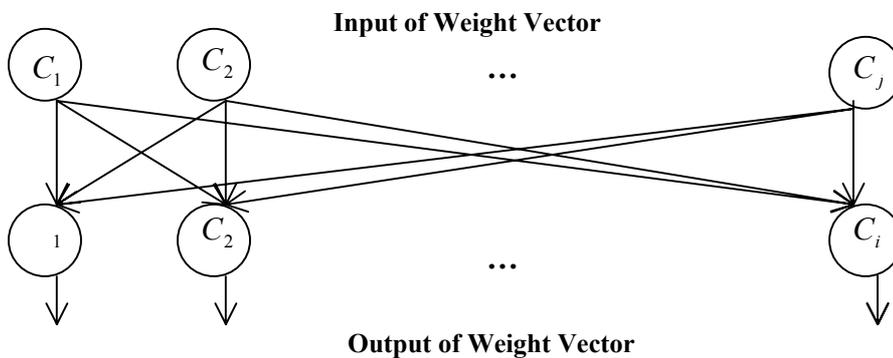


Figure 2 Operation Process of a Learning ANP Model

- (a) The competing criteria (cluster) of a super matrix in Equation (1) have the interaction situation, which is similar to that in Figure 1. The interaction situation of competing criteria is shown in Figure 2 –

which can be regarded as a perfect interaction situation. However, some in-perfect interaction situations (Saaty, 1999) can also be tackled by modifying the relationship among criteria in Figure 2.

- (b) The operation process of the priority with respect to $W_{i,j}$ in Equation (1) is modified from the previous perceptron model as follows:

$$\begin{aligned} w_i &= f(net_i) \\ net_i &= \sum_j r w_{ij} w_j - \theta_i \end{aligned} \quad (4)$$

where

w_j : the weight of the j -th criterion (C_j) in the input level;

w_j : the weight of the j -th criterion (C_j) in the input level;

$r w_{ij}$: the weight of the block matrix W_{ij} for $i \neq j$, it represents the relative importance of the i -th criterion (C_i) over the j -th criterion (C_j); if $i = j$, then $r w_{ij} = 1$; $i, j = 1, 2, \dots, h$;

θ_i : the threshold value of the i -th criterion; if $\sum_j r w_{ij} w_j \geq \theta_i$, then the i -th criterion will be stimulated to be active; otherwise the i -th criterion remained in sleep;

f : the transformation function.

A decision maker can construct the learning ANP by aforementioned assumptions, and the basic learning rule is deduced by minimizing the error function E in each training run:

$$Min E = \sum_i (w_i - w_i^*)^2 \quad (5)$$

where w_i^* denotes the actual w_i . Also assume that $w_i = \sum_j r w_{ij} w_j$; thus, the adjustment factor for each

$r w_{ij}$ is represented by $\eta \times \delta_{ij}$ such that:

$$r w_{ij}^{t+1} = r w_{ij}^t + \eta \times \delta_{ij} \times r w_{ij}^t \quad (6)$$

Where η is a subjectively decided value, which denotes the learning rate and $0 < \eta \leq 1$; t denotes the period; δ_{ij} is given by the following equation in each training run:

$$\delta_{ij} = -\frac{\partial E}{\partial r w_{ij}} = -2 \times (w_i - w_i^*) \times w_j \quad (7)$$

A decision maker can formulate a learning ANP and apply it to predict some useful results after moderately training this learning ANP, e.g., forecasting the short-term market share in Saaty's work (1999) by applying Equation (6) and (7) – if the time series data are available. A simple example is illustrated in the next section to show how this learning ANP works.

3. Numerical Example

The simple learning ANP in this section is assumed by letting Equation (4) satisfy the following conditions:

- (a) $w_i = \sum_j r w_{ij} w_j - \theta_i$;
- (b) $\theta_i = 0$;
- (c) Let $r w_{ij} = 0$ for $i \neq j$ at the beginning with period $t=1$;
- (d) $\eta = 0.5$.

Thus, assume there are three companies, which are symbolized by p_1 , p_2 and p_3 , respectively. The marketing manager in p_1 can rank each company's dominance by his subjectively sales experiences in each period t . Since the market share of each company is resulted from each company's dominance, let us regard the output of the interactions among companies' dominances as the market share for each company. This interaction situation is shown as in Figure 3.

Each company's dominance for input in Figure 3 is computed by Saaty's maximal eigenvalue method after this manager gives a pair-wise comparison matrix of three companies in each period t . The learning ANP is applied such that the sales manager in p_1 is able to combine his subjective sales experiences with a market share prediction model, which can forecast the short-term market share for each company. This sales manager's subjective perception of each company and observed market share are assumed in Table 1.

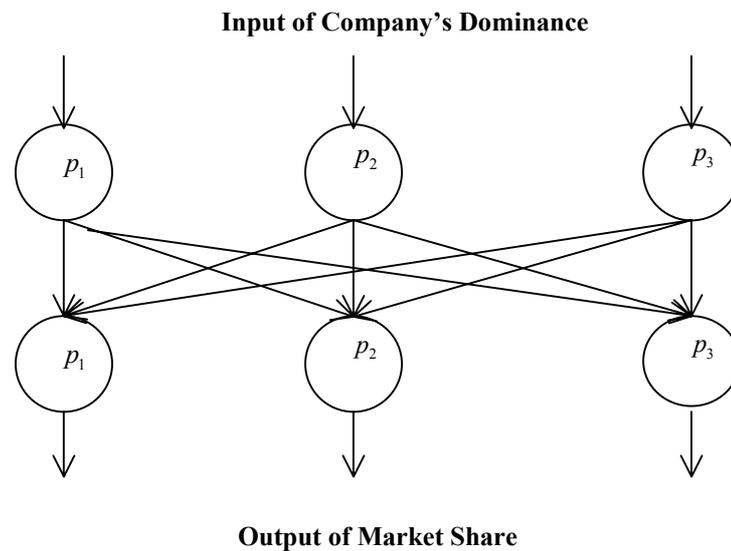


Figure 3 Operation Process of Numerical Example

Table 1 The Sales Manager's Perception and Observed Market Share

Period (t)	1	2	3	4	5	6
Subjective Perception ($w_{p_1}, w_{p_2}, w_{p_3}$)	(0.2,0.5,0.3)	(0.4,0.4,0.2)	(0.2,0.4,0.4)	(0.3,0.3,0.4)	(0.4,0.3,0.3)	(0.2,0.4,0.4)
Observed Market Share ($w_{p_1}^*, w_{p_2}^*, w_{p_3}^*$)	(0.3,0.4,0.3)	(0.5,0.3,0.2)	(0.1,0.5,0.4)	(0.4,0.2,0.4)	(0.4,0.2,0.4)	(0.3,0.4,0.3)

The data from period $t=1$ to period $t=5$ are designed to train the learning ANP, and the data in period $t=6$ is used to check if the subjective input is $(w_{p_1}, w_{p_2}, w_{p_3}) = (0.2, 0.4, 0.4)$ by the manager, then what the market share output will be by this learning ANP. Finally, the computed result derived by learning ANP is

(0.22,0.38,0.40). This result is close to the observed data when period $t=6$. However, the prediction accuracy can be improved as the variety and number of observed data increases (Minsky and Papert, 1969).

4. Conclusions and Recommendations

This paper extends the idea of analytical network process (ANP), which is originally proposed by Saaty (1996) so as to tackle the interactions among many competing criteria when ranking alternatives. It is appropriate to use the learning ANP for minimizing the gap between the subjective perception of a decision maker and the objective reality in practices. For example, this learning ANP can be applied on fault diagnosis of an operation; thus, we can respond early so as to prevent undesired results. The original formulation of a learning ANP is discussed and presented in this study, this learning ANP is able to automatically moderate itself in order to reflect more dynamics as time varies. However, this study only constructs a simple learning ANP with two levels and strong assumptions; thus, it is worthy to explore the feasibility of a multi-level learning ANP by relaxing these assumptions.

References

- Minsky, M. L. and Papert, S. S. (1969), *Perceptrons*, MIT Press, Cambridge, MA.
- Saaty, T. L. (1996), *Decision Making with Dependence and Feedback: The Analytical Network Process*, RWS Publications, Pittsburgh, PA.
- Saaty, T. L. (1999), "Fundamentals of the analytical network process," Proceeding of ISAHP 1999, Aug. 12-14, Kobe, Japan.