A LEAST SQUARES METHOD FOR THE ESTIMATION OF AHP-WEIGHTS FROM AN INCOMPLETE COMPARISON MATRIX

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Abstract

We consider the estimation of weights w_i, $i = 1, \ldots, n$, of n entities from an $n{\times}n$ pairwise comparison matrix $A = [a_{ij}]$ in a situation where only a part of the elements a_{ij} of the matrix is known. In a case where n is large, it may be desirable to avoid making all the $n(n\text{-}1)/2$ possible pairwise comparisons.

The estimates $\hat{w}_i$ of the weights are calculated using the logarithmic least squares method. For a given n and a given number k of pairwise comparisons to be made (where k is not too small but is smaller than $n(n\text{-}1)/2$) we discuss an optimal experimental design, i.e. a choice for the a_{ij} to be assessed that is optimal under suitable criteria.
The statistical analysis is based on the assumption that for each pair (i,j) the logarithm $\log(a_{ij})$ is normally distributed with expectation $\log(w_i/w_j)$ and variance σ^2, common for all (i,j).

REFERENCE: Laininen, P. and Hämäläinen, R. P. : Analyzing AHP-matrices by Regression, http://www.hut.fi/Units/SAL/Publications/m-index.html