A CASE STUDY BY THE METHOD THAT ADDED THE AHP TO LINEAR PROGRAMMING

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Keywords: decision making, linear programming, multi-objective

Summary: *There are several systems to control the air conditioning in a building. Generally, each building has the heat source equipment such as chiller, boiler, hot and chilled water generator, and cooling tower for air cooling and heating. In this case, several heat source facilities are installed in a large-scale building. When a facility manager satisfies the same purpose to keep indoor air quality comfortably, a difference comes out on expense side or environment side (for example, CO_2 discharge) by those operative methods greatly. We can think about this problem as a multi-objective decision making problem, and there is an example solved by using linear programming. However, we cannot deal only with linear programming when objective functions disagree mutually (for example, fuel costs and CO_2 discharge) or when condition of constraints includes a qualitative factor. So in this paper, we introduce the case study that we added the AHP to liner programming and applied in decision making problem of most suitable operative method of the heat source facility.*

1. Introduction

As one of methods for decision making used best, there is liner programming. This method is the model of the decision making problem that is solved by quantitative information. Its structure and relationship among the quantity of the problem is clear. So we can find the optimal solution that maximize or minimize a linear objective function under condition of constraint of some linear equations or some linear inequalities by the linear programming. Generally, benefit, expense or volume of production is selected as a objective function and the aspect of economy, productive activity, resources or the transportation are thought about as a condition of constraint of that time. We show the linear programming problem formularized in a formula (1).

Objective Function:

$$c_1x_1 + c_2x_2 + \cdots + c_nx_n \rightarrow MAX$$

Condition of Constraint:

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m \\ x_1 \geq 0, x_2 \geq 0, \cdots, x_n \geq 0 \end{array}\right\} \quad (1)$$

In this problem we can calculate optimal solution by the simplex method usually. When there is more than one objective function that I showed with an formula (1) here, this problem is called with a multi-objective linear programming problem and calculation of a solution for one objective function is mathematical solution then decision making by consideration of payoff tables by each solution is needed to calculate the optimal solution for it.

In this paper, we introduce the case study that we would let more than one objective function in multi-objective linear programming return to a linear programming problem of one objective by AHP and solved the problem by using linear programming.

2. Multi-objective Programming Problem

When there is more than one objective function in the linear programming problem that showed with an formula (1), we call this with a multi-objective linear programming problem. We use a simple exercise in order to explain the method that added the AHP to linear programming. we show an exercise in a formula (2).

Objective Function:

$$f_1(\mathbf{x}) = 4x_1 + 3x_2 \rightarrow MAX$$
$$f_2(\mathbf{x}) = 0.2x_1 + x_2 \rightarrow MAX$$

Condition of Constraint:

$$\left.\begin{array}{l} 4x_1 + 2x_2 \le 100 \cdots b_1 \\ 3x_1 + 3x_2 \le 90 \cdots\cdots b_2 \\ 2x_1 + 4x_2 \le 100 \cdots b_3 \\ x_1 \ge 0, x_2 \ge 0, \end{array}\right\} \qquad (2)$$

2.1 The Global Evaluation Method

One of solution of a multi-objective linear programming problem includes the global evaluation method. We try to solve the exercise which I showed with a formula (2) by the global evaluation method.

At first the optimal solution is P (20,.10) and the maximum of $f_1(\mathbf{x})$ becomes 110 when we paid out attention to only $f_1(\mathbf{x})$ <case 1>.
Next the optimal solution is Q (0,.25) and the maximum of $f_2(\mathbf{x})$ becomes 25 when we paid out attention to only $f_2(\mathbf{x})$ <case 2>. We summarize the above-mentioned result in Table 1.

Table 1 Payoff table

	$\mathbf{x}^*$		$f_1(\mathbf{x}^*)$	$f_2(\mathbf{x}^*)$
	x_1	x_2		
case 1	20	10	**110**	(14)
case 2	0	25	(75)	**25**

It is necessary for decision maker to pursue only one solution from the relation that is showed by above payoff table. With the global evaluation method, we make a new objective function that minimize the sum of relative deviation S_k of each maximum $f_k(\mathbf{x}^*)$ with optimal solution $\mathbf{x}^*$ between each objective function $f_k(\mathbf{x})$ and we grind the optimal solution which cleared it up with an answer of a multi-objective linear programming problem as a compromise solution. We show the global evaluation method formularized in a formula (3).

Objective Function:

$$f(\mathbf{x}) = \sum_{k=1}^{l} S_k \rightarrow MIN$$

Here, (3)

$$S_k = \frac{f_k(\mathbf{x}^*) - \sum_{j=1}^{n} c_{kj} x_j}{f_k(\mathbf{x}^*)} \qquad (k = 1, 2, \cdots, l)$$

We show the result that applied each optimal solution which we showed by Table 1 in a formula (3) with a formula (4).

Objective Function:

$$f(\mathbf{x}) = \frac{110 - (4x_1 + 3x_2)}{110} + \frac{25 - (0.2x_1 + x_2)}{25} \rightarrow MIN \tag{4}$$

A compromise solution becomes R (10, 20) and $f_1^*(\mathbf{x})=100$, $f_2^*(\mathbf{x})=22$ when we solve a formula (4). We show the above-mentioned relation with Figures 1 and 2.

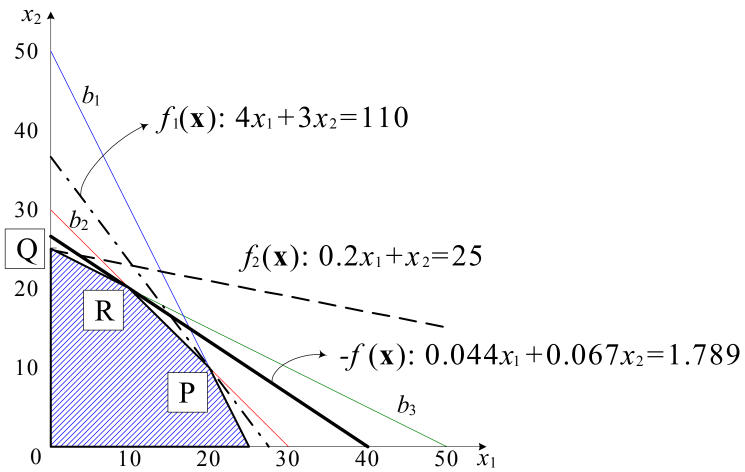

Figure 1 Calculation of a compromise solution R

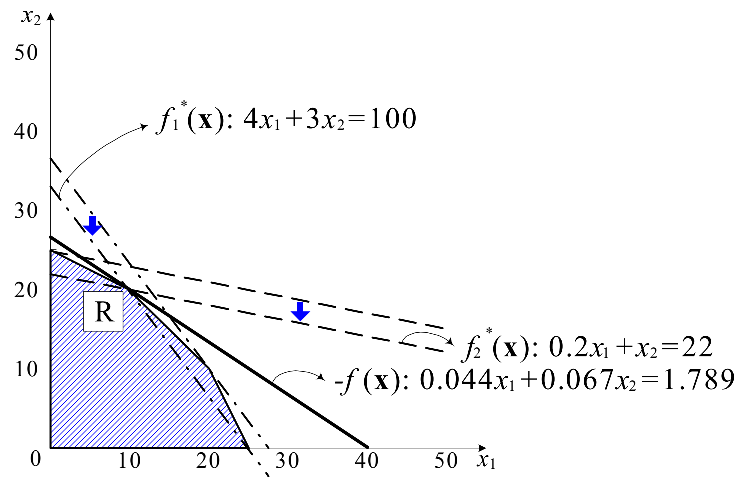

Figure 2 A substitution result of a compromise solution R

In case 1, an optimal solution is P(20, 10) and then $f_1(\mathbf{x}^*)$ becomes 110 and $f_2(\mathbf{x})$ becomes 10. In addition, in case 2, an optimal solution is Q(0, 25) ant then $f_1(\mathbf{x})$ becomes 75 and $f_2(\mathbf{x}^*)$ becomes 25. However, in case of the global evaluation method, a compromise solution is R(10, 20) ant then $f_1^*(\mathbf{x})$ becomes 100 and $f_2^*(\mathbf{x})$ becomes 22.

2.2 When we let more than one objective function return to one by the AHP

This method considers the weight between more than one objective functions objectively and integrates them in one objective function. And we suggest that we used the AHP on the occasion of weighting. We show a objective function formularized at that time in a formula (5).

Objective Function:

$$g(\mathbf{x}) = \sum_{k=1}^{l} \alpha_k f_k(\mathbf{x}) \rightarrow MAX \tag{5}$$

Here,

$$\alpha_k \geq 0$$

We consider the exercise that we showed with a formula (2) by this method. We become a formula (6) when we apply a formula (5) in the exercise. But we do it with a condition of $\alpha_1+\alpha_2=1$ and $\alpha_1=\alpha$.

Objective Function:

$$g(\mathbf{x}) = \alpha(4x_1 + 3x_2) + (1-\alpha)(0.2x_1 + x_2) \rightarrow MAX$$

Condition of Constraint:

$$\left.\begin{array}{l} 4x_1 + 2x_2 \leq 100 \cdots b_1 \\ 3x_1 + 3x_2 \leq 90 \cdots\cdots b_2 \\ 2x_1 + 4x_2 \leq 100 \cdots b_3 \\ x_1 \geq 0, x_2 \geq 0, \end{array}\right\} \tag{6}$$

We show results at having let α in a formula (6) change into {0, 0.1, ･･･ , 1} with Table 2.

Table 2 A change of a solution by α

α	$g(\mathbf{x})$	$\mathbf{x}^*$		$g(\mathbf{x}^*)$	$f_1(\mathbf{x}^*)$	$f_2(\mathbf{x}^*)$
		x_1	x_2			
0.0	$4.00x_1+3.00x_2$ $(=f_1(\mathbf{x}))$	20	10	110.0	110	14
0.1	$3.62x_1+2.80x_2$	20	10	100.4	110	14
0.2	$3.24x_1+2.80x_2$	20	10	90.8	110	14
0.3	$2.86x_1+2.80x_2$	20	10	81.2	110	14
0.4	$2.48x_1+2.80x_2$	20	10	71.6	110	14
0.5	$2.10x_1+2.80x_2$	20	10	62.0	110	14
0.6	$1.72x_1+2.80x_2$	10	20	53.2	100	22
0.7	$1.34x_1+2.80x_2$	10	20	45.4	100	22
0.8	$0.96x_1+2.80x_2$	10	20	37.6	100	22
0.9	$0.58x_1+2.80x_2$	0	25	30.0	75	25
1.0	$0.20x_1+2.80x_2$ $(=f_2(\mathbf{x}))$	0	25	25.0	75	25

In addition, we show an appearance of a change of $g(\mathbf{x})$ with Figure 3.

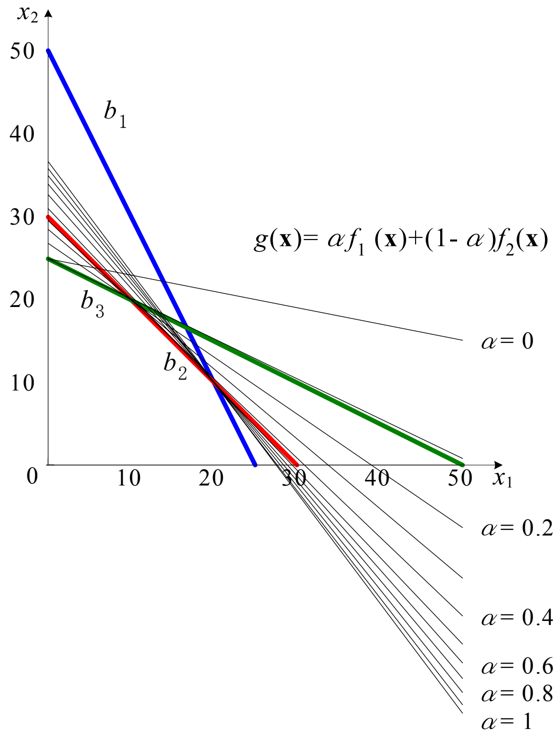

Figure 3 A change of $g(\mathbf{x})$ by α

In this method, a degree of leaning of $g(\mathbf{x})$ moved at an interval of a degree of leaning of $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ by a value of α and, as a result, was able to get three kinds of solutions. In case of α =0.6, 0.7, 0.8, we get a solution the same as a compromise solution by the global evaluation method that I showed with 2.1. In other words it can be said that the global evaluation method is equal in case of the weight by this method is the middle relatively. Furthermore, in case of this method, we can get a solution at having regarded $f_1(\mathbf{x})$ or $f_2(\mathbf{x})$ as important more. And by this method, we can let we are more flexible and reflect subjectivity of a decision maker compared with the global evaluation method.

3. A case study

We introduce the case that applied this method in a problem of most suitable use of the heat source equipment in one building.

3.1 The target model

We show a total figure of a system including the heat source facility that we intended for in Figure 4. The heat source facility is machinery making chilled or hot water for air conditioning and sanitation as energy with electricity and gas, but as for them, there is plural number level and it is connected with each other including supporting machinery such as a pump or a cooling tower complicatedly as we show it in Figure 4. For example, it is a very difficult problem to turn a driving cost of the heat source facility into a minimum while satisfying environment to a resident in such situation.

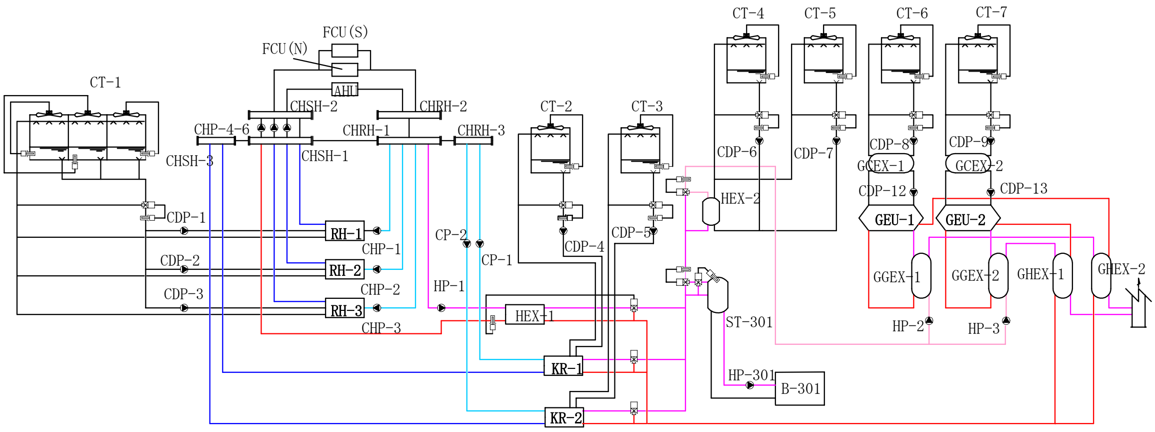

Figure 4 A total figure of a system including the heat source facility

In addition, we show an outline of main heat source machinery in Figure 4 with Table 3.

Table 3 Outline of main heat source machinery

<table>
<tr><th rowspan="2">Sign</th><th rowspan="2">Name</th><th colspan="4">Function</th></tr>
<tr><th colspan="2">Input</th><th colspan="2">Output</th></tr>
<tr><td>RH-1</td><td rowspan="5">Hot and chilled water generator</td><td rowspan="3">City gas and electricity</td><td rowspan="2">City gas: 68.2 (Nm^3/ h)
Electricity: 12.5 (kW)</td><td rowspan="5">Chilled water (C. W.) or hot water (H. W.)</td><td rowspan="2">C. W. : 756.0 (Mcal/ h)
H. W.: 599.0 (Mcal/ h)</td></tr>
<tr><td>RH-2</td></tr>
<tr><td>RH-3</td><td>City gas: 29.4
Electricity: 6.5</td><td>C. W. : 302.4
H. W. : 262.0</td></tr>
<tr><td>KR-1</td><td rowspan="2">Electricity and high temperature hot water (from GEU)</td><td rowspan="2">Electricity: 8.5</td><td rowspan="2">C.W. :241.9</td></tr>
<tr><td>KR-2</td></tr>
<tr><td>GEU-1</td><td rowspan="2">Co-generator with gas engine</td><td rowspan="2">City gas and electricity</td><td rowspan="2">City gas: 68.0
Electricity: 62.5</td><td rowspan="2">Electricity and high temperature hot water</td><td rowspan="2">H.W. :172.5
Electricity: 250</td></tr>
<tr><td>GEU-2</td></tr>
</table>

As a whole, this system does electricity and chilled or hot water for air conditioning and sanitation needed in the building with the output from gas and high temperature hot water as input as you understand it form Figure 4 and Table 3. In addition, they can purchase it in case of electricity directly from an electric power company.

3.2 Confirmation of pattern of electricity load and an air conditioning load

On electricity and the chilled and hot water which are the output of this system, it is necessary to confirm needed quantity (electricity load and air conditioning load) actually. This becomes a condition of constraint, but it changes greatly by a season or a period of time. As an example, we show electricity load and an air conditioning load pattern of one day with Figure 5.

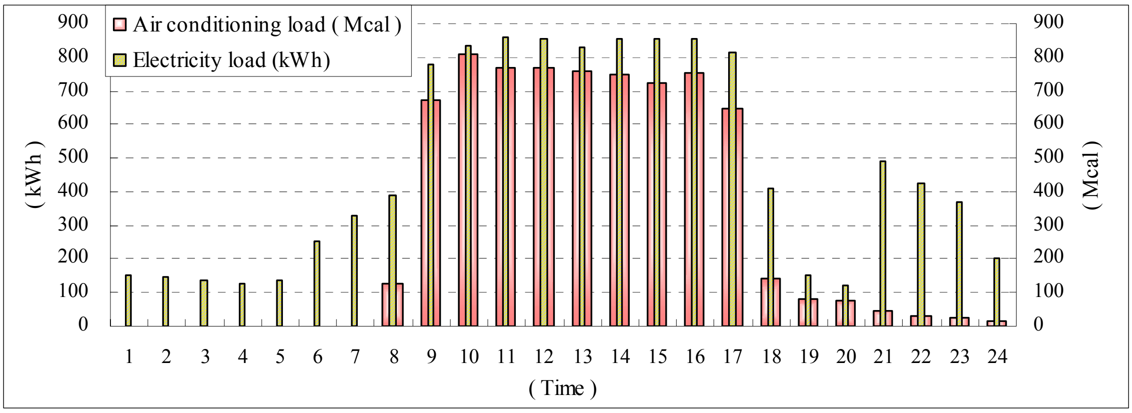

Figure 5 Electricity load and an air conditioning load pattern of one day

3.3 Formularization of the target model and a result of calculation

We consider the economic side and environmental problem and do that we minimize a driving cost and a CO_2 discharge of heat source facility with a purpose. We show this problem formularized with a formula (7).

Objective Function

at turning a driving cost into a minimum:

$$2105.7\,x_1 + 2105.7\,x_2 + 924.0\,x_3 + 2954.2\,x_4 + 2954.2\,x_5 + 17.8\,x_6 \rightarrow MIN$$

at turning a CO_2 discharge into a minimum:

$$161.2\,x_1 + 161.2\,x_2 + 70.0\,x_3 + 181.9\,x_4 + 181.9\,x_5 + 0.4\,x_6 \rightarrow MIN$$

Condition of Constraint: (7)

$$756.0\,x_1 + 756.0\,x_2 + 302.4\,x_3 + 241.9\,x_4 + 241.9\,x_5 \geq Q$$

$$250\,x_1 + 250\,x_2 + x_6 \geq E$$

Here,

x_1: use ratio of RH-1, x_2: use ratio of RH-2, x_3: use ratio of RH-3
x_4: use ratio of GUX-1 and KR-1, x_5: use ratio of GUX-2 and KR-2
x_6: quantity of purchase electricity
Q: Air conditioning load, E: Electricity load

By the way, if facility manager in the building think about the weight of minimization of cost as α and the weight of minimization of CO_2 discharge as (1- α) here, two objective function in a formula (7) become a formula (8).

Objective Function:

$$(1944.5\,\alpha + 161.2)\,(x_1 + x_2) + (854.1\,\alpha + 69.9)\,x_3 + (2772.3\,\alpha + 181.9)\,(x_4 + x_5) + (17.5\,\alpha + 0.4)\,x_6 \rightarrow MIN \quad (8)$$

Actually, it is necessary to calculate for a load pattern at every hour. We show the result in case having let α in a formula (8) change at 9:00 in Figure 5 with Table 4.

Table 4 Outline of main heat source machinery

Priority	CO_2 discharge minimum ⇔ Cost minimum		
α	[0, 0.018]	(0.018, 0.057]	(0.057, 1]
Optimal solution	x_1=0.888 x_6=779 Others are 0.	x_4=2.773 x_6=85.6 Others are 0.	x_4=3.116 Others are 0
Cost	15,766	9,721	9,205
CO_2 discharge	424	535	567
Realistic driving method of the heat source	Use RH-1 with precedence for air conditioning load and purchase electricity for its load.	Use GEU-1 and GEU-2 with precedence for air conditioning load and electricity load.	Use GEU-1 and GEU-2 with precedence for air conditioning load and electricity load.

We got two ways as a realistic operative method but there were three solutions by a value of α. In other words a value of α is the weight between cost and CO_2 discharge that a decision maker thinks about.

4. Conclusion

We suggested a method to let more than one objective function return to one by a weight charge account by the AHP of a multi-objective linear programming problem and introduced the case that applied this method to a problem of most suitable usage of heat source facility. As you understand it from Table 4, when α which is a driving method to minimize CO_2 discharge is equal to or less than 0.018. α=0.018 means that the facility manager thinks that CO_2 discharge is more important 55 times than cost. With the present method, a result is controlled greatly by an absolute quantity of a coefficient of the same variable between different objective functions. We intend to improve this point in the future.

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